June 18, 2022

ICE, CLOUD, AND LAND ELEVATION SATELLITE-2 (ICESat-2)

Algorithm Theoretical Basis Document (ATBD)
for
ATL19/23
Gridded Dynamic Ocean Topography

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This document may be cited as:

Goddard Space Flight Center
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Abstract

This document describes the theoretical basis of the ocean processing algorithms and the products that are produced by the ICESat-2 mission. It includes descriptions of the parameters that are provided in each product as well as ancillary geophysical parameters, which are used in the derivation of these ICESat-2 products.
ICESat-2 Algorithm Theoretical Basis Document for Gridded Dynamic Ocean Topography
Release 003(ATL19)/Release 001(ATL23)
CM Foreword

This document is an Ice, Cloud, and Land Elevation Satellite-2 (ICESat-2) Project Science Office controlled document. Changes to this document require prior approval of the Science Development Team ATBD Lead or designee. Proposed changes shall be submitted in the ICESat-II Management Information System (MIS) via a Signature Controlled Request (SCoRe), along with supportive material justifying the proposed change.

In this document, a requirement is identified by “shall,” a good practice by “should,” permission by “may” or “can,” expectation by “will,” and descriptive material by “is.”

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ICESat-2 Algorithm Theoretical Basis Document for Gridded Dynamic Ocean Topography

Release 003(ATL19)/Release 001(ATL23)
Preface

This document is the Algorithm Theoretical Basis Document for the processing open ocean data to be implemented at the ICESat-2 Science Investigator-led Processing System (SIPS). The SIPS supports the ATLAS (Advance Topographic Laser Altimeter System) instrument on the ICESat-2 Spacecraft and encompasses the ATLAS Science Algorithm Software (ASAS) and the Scheduling and Data Management System (SDMS). The science algorithm software will produce Level 0 through Level 4 standard data products as well as the associated product quality assessments and metadata information.

The ICESat-2 Science Development Team, in support of the ICESat-2 Project Science Office (PSO), assumes responsibility for this document and updates it, as required, as algorithms are refined or to meet the needs of the ICESat-2 SIPS. Reviews of this document are performed when appropriate and as needed updates to this document are made. Changes to this document will be made by complete revision.

Changes to this document require prior approval of the Change Authority listed on the signature page. Proposed changes shall be submitted to the ICESat-2 PSO, along with supportive material justifying the proposed change.

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ICESat-2 Algorithm Theoretical Basis Document for Gridded Dynamic Ocean Topography

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*** Signatures are available on-line at: https://icesatiimis.gsfc.nasa.gov ***
# Change History Log

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<td>Initial Release</td>
<td>(11/9/2020 – Section 5.6.3.2.1 regarding equation 48 description, changed “cross product” to “product sum”) The last ATL12 ATBD with a complete ATL19 description is ICESat2_JMdraft_Ocean_atbd_12012020_SD dated Jan. 4, 2021. Changes to ATL19 prior to 11/09/2020 are included in that ATL12 ATBD dated 12/30/2020, ICESat2_JMdraft_Ocean_atbd_12302020_CX. Changes to ATL19 ATBD from 12/3/2020 through 12/30/2020 were not tracked or logged and this ATBD originating as Morison’s: ATL12 ATBD ICESat2_JMdraft_Ocean_atbd_12302020_CX should be considered the original ATL19 ATBD, 02/04/2021 Globally corrected grid_lon and grid_lat to lon_avg and lat_avg 02/04/2021 Corrected the anomaly equations at the end of the first paragraph of Appendix B by multiplying by (1/N) 02/22/2021 Finish global change of dof_grid to dof 02/22/2021 Section 3.2.4.2 correct dot_sigma_dfw_albm to dot_sigma_dfwalbm 02/22/2021 global change of dot_dfwallbeam to dot_dfwalbm 02/22/2021 global change of dot_dfw_uncertain to dot_dfw_uncrtn 02/22/2021 Section 3.2.3.1.1 added computation of uncertainty in simple averages of DOT: “To compute the uncertainty, dot_avg_uncrtn, in gridded DOT, dot_avg, divide dot_sigma_avg by the square root of dof to establish the uncertainty in the degree-of-freedom weighted DOT.” Also added dot_avg_uncrtn to Table 2 02/25/2021 In Table 2 corrected description of dot_avgcntr to “Simple average of dynamic ocean topography interpolated center of grid cell” 03/01/2021 In equation 49 corrected the equation for C to</td>
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</table>
04/14/2021 to 4/26/2021 – Numerous small corrections to make variable names consistent throughout

04/21/2021 – Section 3.2 Reordered and expanded to:
3.2 Gridding DOT and SSH for ATL19
   3.2.1 The Grids
   3.2.2 Temporal Averaging
   3.2.3 Input to Gridding
   3.2.4 Gridding
   3.2.5 Gridding Output

and

Expand Introduction to include possible ice parameters and optimal interpolation considerations:
1.0 INTRODUCTION and Background
1.1 Background: ATL03 and ATL12
1.2 ATL19 Gridded Product
   1.2.1 ATL19 Grids
   1.2.2 The Basic Product
   1.2.3 The running 3-month average
   1.2.4 Merging with ATL10 to produce global DOT
   1.2.5 Optimal interpolation of DOT

04/23/2021 Table 2 – Added “_albm” variables

04/25/2021 Added Appendix E: Optimal Interpolation of ICESat-2 Dynamic Ocean Topography

4/29/2021 Numerous edits for clarity in response to David Hancock’s comments

04/30/2021 – Added Figures 2-6 and related discussions
   - Changed Table 2 to Table 3
   - Added a new Table 2 listing inter-beam biases

6/23/2021 Section 3.2.3.1 Pre-grid filtering was changed to a single pass over ATL12 ocean segments rejecting any ocean segment with average DOT departing from the 10-degree latitude, all-ATL12 average DOT by more than three times the all-ATL12 standard deviation of DOT.

9/3/2021 Completed numerous edits suggested by reviewers Laurie
Padman and Steve Nerem. This included adding ATL12 background and ATL19 rationale in revised introduction. The noteworthy conceptual addition not mentioned by reviewers was clarifying that the higher moments and uncertainty that are gridded represent only the sea state induced variances and that determining ocean segment-to-segment variability should be addressed with TBD methods.

9/4/2021 Added *meanoffit2* and *ds_y_bincenters* to Table 1 Inputs from ATL12 and added *dot_hist*, *dot_hist_albm*, and *ds_y_bincenters* to Table 3 Outputs

9/4/2021 Changed *dot_hist_grid* to *dot_hist* throughout and changed description of calculation of *dot_hist* to:

“To compute the bin aggregate probability density function (PDF), *dot_hist*, of DOT, we first must convert each *Y* PDF from ATL12 to a PDF of DOT by adding *meanoffit2* to the x-axis of *Y*, *ds_y_bincenters* and then interpolating the result to an intermediate PDF, *Y_intermediate*, evaluated at the original *ds_y_bincenters*. (Note: In ATL19 Release 1, meanoffit2 was inadvertently not added so the aggregate histograms only reflect the aggregate wave environment with mean near zero). The aggregate probability PDF, *dot_hist*, of DOT will equal the sum *Y_intermediate x n_photons* in each histogram bin of all *Y_intermediate* divided by the total, *n_photons_gridttl*, of all *n_photons.*”

2/8/2022 In Section 3.2.3 changed the DOT to be averaged for all products to include the sea state bias correction equal to subtracting the ATL12 ocean segment average of SSB, i.e., dot=h-geoid_seg-bin_ssbias

2/8/2022 Modified Section 3.2.4.1 and Table 3 to:

1) Remove “cntr” averages of depth, geoid and SWH and change *ssb_avgcntr* to a value interpolated to the center using the *a, b, c* coefficients for *dot_avgcntr*, now called *a_avg, b_avg, and c_avg*
2) Called for saving and outputting *a_avg, b_avg, and c_avg*
3) Modified the editing criteria to comparing *QoF* to the departure of the 9-cell average of DOT from DOT interpolated to the average of ocean segment positions in the 9 cells

2/8/2022 Modified Section 3.2.4.4.2 and Table 3 to:
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1) Remove “cntr” dfw averages of depth, geoid and SWH and change ssb_dfwcntr to a value interpolated to the center using the a, b, c coefficients for dot_dfwcntr, now called a_dfw, b_dfw, and c_dfw.

2) Called for saving and outputting a_dfw, b_dfw, and c_dfw
Modified the editing criteria to comparing QoF to the departure of the 9-cell dfw average of DOT from DOT interpolated to the dfw average of ocean segment positions in the 9 cells.

2/11/2022 Edited 3.2.4.4.1 and 3.2.4.4.2 To eliminate variable names that would duplicate exiting variable names and worked to clarify the procedure to calculate dfw and average center values.

02/11/2022 In 3.2.4.4.1 and 3.2.4.4.2 changed the derivation of ssb_avgcntr and ssb_dfwcntr to the difference between DOT calculated with and without the SSB correction.

02/14/2022 In 3.2.4.4 first paragraph, changed:
a minimum of three ocean segments in the cell to compute a center-interpolated DOT value.” to
“a minimum of four ocean segments in the cell to compute a center-interpolated DOT value.”

2/17/2022 In 3.2.4.4.1.1 and 3.2.4.4.2.2 corrected all a coefficients to multiply times longitude and b coefficients times latitude. Also DOT-avgentrnobias corrected to DOT_avgentrnobias and DOT-dfwcntnobias corrected to DOT_dfwcntnobias.

2/24/2022 In 3.2.4.4.1.1 and 3.2.4.4.2.2 changed the editing criteria for center values to
Edit the centered average values according to the following criteria related to the variation of DOT across the cell and the quality of the linear fit. For center cells with an average DOT, dot_avg_albm, remove values of dot_avgcntr when

\[
\text{abs} \left( \text{dot\_avg\_albm} - (a\_avg*\text{lon\_avg} + b\_avg*\text{lat\_avg} + c\_avg) \right) > 2\times QoF \tag{51}
\]

where \( QoF = \text{RMS} (\text{DOT\_seg} - a\_avg*\text{lon\_seg} + b\_avg*\text{lat\_seg} + c\_avg) \),
and

Edit the centered average values according to the following criteria related to the variation of DOT across the cell and the quality of the linear fit. For center cells with an average DOT, \textit{dot\_dfw\_albm}, remove values of \textit{dot\_dfw\_cntr} when

\[
\text{abs} \left( \text{dot\_dfw} - (a\_dfw*\text{lon\_dfw} + b\_dfw*\text{lat\_dfw} + c\_dfw) \right) > 2*QoF
\]

where \(QoF = \text{RMS} \left( \text{DOT\_seg} - a\_dfw*\text{lon\_seg} + b\_dfw*\text{lat\_seg} + c\_dfw \right)\),

2/24/2022 In 3.2.4.4.1.1 and 3.2.4.4.2.2 edited to specify that centered values would be computed using all beams in the 9-cell cluster.

2/24/2022 In Table 3 specified centered average and dfw averaged sea state bias would be calculated as specified in the ATBD.

2/24/2022 In Table 3 eliminated \textit{dot\_avgcntr\_albm}, \textit{dot\_dfwcntr\_albm}, \textit{ssb\_avgcntr\_albm}, and \textit{ssb\_avgcntr\_albm} as redundant with \textit{dot\_avgcntr}, \textit{dot\_dfwcntr}, \textit{ssb\_avgcntr}, and \textit{ssb\_avgcntr} computed with all-beam data.

3/8/2022 Added ice concentration, ice\_conc, to variables such as depth to be averaged in single beam and all beam data. Added to Table 1 Inputs, Section 3.2.4.2.1, and Table 3 Outputs

3/8/2022 Where the criteria for editing “\_cntr” interpolated values for grid cells without a dot\_avg had been TBD, changed the criteria to “\(QoF\) is greater than 0.2 m, a value estimated from examination of distributions of \textit{OoF} to exceed more than 95% of \textit{QoF} values”

4/5/2022 Corrected Table 3 for consistent treatment of variable names and consistency with ATL19 data files: Added “\_”\_albm to \textit{dot\_dfw\_uncrtn} in Table3
Added "_"_albm to dot_hist and deleted row for dot_hist_albm in Table 3 consistent with formatting of other albm variable mentions in the table.

Corrected name surf_prct_avg_albm in Table 3 consistent with formatting of other albm variable mentions in the table.

Added row for surf_prct_dfw and surf_prct_dfw"_"_albm to Table 3.

Added popped_flag variables to Table 1 and Table 3 and a short paragraph at the beginning of Section 3.2.3.1:

"The ATL12 ocean segment data going into ATL19 are already filtered for depths greater than 10-m and for pointing and orbit determinations outside of nominal conditions (ATL03 podppd flag = 0, nominal, or 4, nominal calibration maneuver). Ocean segment averages of depth, depth_seg, and the highest of the podppd flag used in an ocean segment, podppd_flag_seg, are included in ATL12 output. ATL19 gridded averages of these quantities are computed as discussed below and in Table 3."

Deleted the sea ice flag row and TBD from Table 3 because we now have ice concentration.

4/20/2022 In table 3, modified the description of surf_type_prct to better explain it as: "The percentages of each surf_type of the photons in the ocean segment as a 5-element variable with each element corresponding to the percentage of photons coming from positions under each of the 5 surface masks. Due to mask overlaps, photons can originate from more than one mask type, and the 5 surface type percentages can total more than 100%.

"5/15/2022 - In Section 3.2.4.4.1 added calculation of uncertainty in DOT average at the cell center position, dot_avgcntr_uncrt, based on temporary appendix F and specified dot_avgcntr_uncrt as the editing criteria for center values in place of QoF. Also add dot_avgcntr_uncrt to Table 3, and added Appendix F. Appendix F can be substituted for Appendix C once dot_avgcntr_uncrt is successfully implemented and checked.

5/17/2022 – In section 3.2.4.4.1 corrected parentheses mistake in
equation for $QoF$ and simplified editing to discard any $dot\_avgcntr$ values for which the $dot\_avgcntr\_uncrtn$ exceeds TBD $m = 0.2m$ in initial tests.

5/31/2022 – In Section 3.2.4.4.1, equation 14F, corrected parentheses mistake and substituted $L_{xx}$ for $L_{yy}$ in the uncertainty of $b$ term. Other small edits for clarity.

6/16/2022 – Incorporated the uncertainty calculation portion of what had been **Appendix F** into **Appendix B** and changed the normalization of (B8), which had been (F8) from $1/(N-3)$ to $1/(N-2)$. In the body of the ATBD the calculation of $QoF$ was changed to reflect the uncertainty as calculated according to (B8), and numerous formatting and minor editorial changes were also made.

6/16/2022 – Also in 3.2.4.4, added a paragraph justifying the inclusion of a land mask, *landmask*, equal to 0 to indicate if a cell center point was on land. Also added *landmask* to the output table, Table 3.

6/17/2022 – Discussion surrounding equation (55) grayed out pending development of uncertainty for dfw variables. Hierarchy, Page 42, (under ATL12 inputs to ATL19), changed SSH-geoid_seg to SSH-geoid_seg-bin_ssbias.

Page 42 (3.2.4.4.1 and 3.2.4.4.2) Removed the centered values we no longer calculate, keeping only dot and ssb.

Page 42 (3.2.3.2.2), changed grid_lat_dfw to lat_dfw

Page 42 (3.2.3.2.2), changed grid_lon_dfw to lon_dfw

Page 43 (3.2.4.2.1), changed surf_avg to surf_prcnt_avg

6/25/2022 – Changed variable name *land_mask* to *landmask* throughout

**Not Implemented in Rel. 03, V. 001** - 6/25/2022 – In Appendix B, added with editing for style section B.2 Degree-of-Freedom-Uncertainty Weighted Average DOT from John Robbins write up of weighted least square. 6/21/2022.)

**Not Implemented in Rel. 03, V. 001-** 6/25/2022 – Changed 3.2.4.2.2 to Averaging Weighted by Degrees-of-Freedom-Uncertainty and changed all the dfw averages from averages weighted by $DOF = np\_effect$ to averages weighted by $Wi = (1/h\_uncrtn)^2$. Also added explanation that because this weight is equal to $np\_effect/dot\_sigma$ it includes the effect of wave
amplitude as well as degrees-of-freedom on uncertainty, and the resulting averages weighted by $Wi$ should have the minimum uncertainty. The $dfw$ variable names were not changed but are referred to generically in the text as degree-of-freedom-uncertainty weighted variables (same acronym, slightly different meaning))

(Not Implemented in Rel. 03, V. 001 - 6/25/2022 – Made separate Section 3.2.4.4.2 for centered dfw averages weighted Wi and including uncertainty for the dot_dfwcntr values according to the Robbins write up in Appendix B.2.)

6/28/2022 - Numerous edits suggested by John Robbins and David Hancock to identify ATL23 as the 3-month moving average gridded product.

7/12/2022 – Incorporated edits by Suzanne Dickinson combining the ATBD dated 6/17/2022 used as a basis for the Rel. 003, V. 001 ASAS software with the 6/28/2022 version. The uncertainty weighted dfw center fit and associated uncertainty are not included. Ice concentration averages output, $ice\_conc\_avg$, name has been contracted to $ice\_conc$ in 3.2.4.2.1 consistent with Table 3.

7/29/2022 – In section 3.2.4.2.1 modified description of the aggregate histogram calculation and definition of $dot\_hist$ to include $ds\_hist\_bincents$ thusly: $dot\_hist$ extends over bins centered $ds\_hist\_bincents$ that are the same as $ds\_y\_bincents$ truncated to ±3m. (Note: In ATL19 Release 2, $ds\_hist\_bincents$ is referred to as $ds\_grid\_dot$.)

8/4/2022 – Correction to the above changed: “truncated to ±3m. (Note: In ATL19 Release 2, $ds\_hist\_bincents$ is referred to as $ds\_grid\_dot$)” to: “, i.e., ±15m”

8/13/2022 – In Table 1 moved $ds\_y\_bincents$ to the top of the table and eliminated and deleted $ancillary\_data$ group.

8/13/2022 – Added to Table 3 $ds\_hist\_bincents$ with mention of ($ds\_grid\_dot$ in Release 2).

8/4/2022 – Deleted mention of “($ds\_grid\_dot$ in Release 2).”

8/13/2022 – Added to the Table 3 description of $dot\_hist$ and $dot\_hist\_albm$,”Histogram bin centers are given by $ds\_hist\_bincents$. ($ds\_grid\_dot$ in Release 2)”
8/13/2022 – In Table 3: Deleted mention of “(ds_grid_dot in Release 2).”
8/13/2022 – In Table 3: Moved variable dimension for bins in dot_hist_albm to first position.
8/13/2022 – In Table 3: To reduce data storage, eliminate dot_hist, keeping only dot_hist_albm.

8/13/2022 – At the end of Section 3.2.4.2.1: Added the paragraph to say only computing dot_hist_albm,

“Note that in the implementation of Release 2 of ATL19, we found that including dot_hist for each individual, beam each grid cell, and 6001 histogram bins used an excessive amount of storage necessitating the computation of dot_hist only as an aggregate for all beams termed dot_hist_albm.“

8/13/2022 – In Table 3: Removed dot_hist leaving only dot_hist_albm with description: “All beam aggregate probability density function of DOT histograms (ATL12 Y histogram + measnoffit2). Histogram bin centers are given by ds_hist_bincenters for each of the three grids.

8/15/2022 – In Table 3 eliminate “Not implemented in Rel 3” statement for dot_dfw_uncrtn.

8/15/2022- At end of 3.2.4.2.2 replaced TBD with using least-squares weighted by a function of h_uncrtn.
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Release 003(ATL19)/Release 001(ATL23)

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1.0 INTRODUCTION AND BACKGROUND

This ATBD will cover the gridding of dynamic ocean topography and related variables from ICESat-2 ATL12 sea surface height (SSH).

1.1 Background: ATL03 and ATL12

The Ice, Cloud and land Elevation Satellite 2 (ICESat-2) is a photon-counting pulsed laser altimeter intended primarily to map the heights of the Earth’s ice and snow-covered and vegetated surfaces. Its Advanced Topographic Laser System (ATLAS) projects 3 pairs of strong and weak beams pulsed at 10 kHz. For each beam, it measures the time of flight of individual photons to the Earth’s surface and back. This range combined with precision pointing and orbit determination is used to measure the height of the surface along ground tracks numbered from left to right (gt1L, gt1R, gt2L, gt2R, gt3L, and gt3R) across the path of ICESat-2 (Fig. 1 right). The 6 beams are arranged in 2 rows of 3 with the weak beams forward when flying in the forward direction. The track assignments of the beams are as shown in Figure 1 (when during half the year ICESat-2 is flying backward the ground tracks remain numbered left to right but the beam assignments flip left to right). With the spacecraft yawed slightly to the left the weak and strong beam tracks of each pair are separated by only 90 m with the tracks of the strong-weak pairs separated 3 km across track. Each pulse of each beam illuminates a patch on the surface about 14 m across, and with the spacecraft moving at 7 km s\(^{-1}\), new patches are illuminated every 0.7 m, giving ICESat-2 unparalleled along track spatial resolution. The orbit of ICESat-2 extends to North and South 88° to capture the Polar Regions and repeats every 91 days.

Although ICESat-2 was not intended primarily as an ocean altimeter, its fine resolution and polar reach make it a uniquely exciting ocean instrument. Consequently, the ICESat-2 ATL12 along track ocean surface height product has been developed (Morison et al., 2019). It draws input data mainly from the ICESat-2 ATL03 Global Geolocated Photon heights product.

The ICESat-2 ATL03 [Neumann et al., 2021a, 2021b] provides the photon reflection height (referred to as photon height) of the ocean surface relative to the WGS84 ellipsoid for the downlinked data of each of the 6 beams. Originally, over the ice-free ocean and away from land only the strong beam data were downlinked to conserve downlink data volume, and weak beam data was only acquired over the ocean when near land or over sea ice. Beginning in the summer of 2021, this limitation has
been relaxed so that strong and weak beam data everywhere is downlinked from the satellite. The raw photon heights are corrected for atmospheric delay and standard geophysical corrections such as solid earth tide of common concern to all the higher-level ICESat-2 products (e.g., land ice, sea ice and vegetation). A statistical approach is used to assign a confidence rating to the likelihood of each photon height being a surface height.

The processing of the ATL03 photon heights to produce the ATL12 ocean surface height [Morison et al., 2019] first involves removing from the photon heights the expected high frequency variations due to tides from the GOT4.8 model and short period atmospheric forcing with a Dynamic Atmospheric Correction (DAC) based on the 6-h AVISO MOG2D. To further reduce the height variability of the raw photon heights, the EGM2008 geoid in the mean tide system is subtracted, so that processing begins with photon heights expressed as dynamic ocean topography (DOT) dealiased for tides and short period atmospheric forcing. The processing system then accumulates histograms of these dealiased and geoid-referenced photon surface heights along each ground track over ocean segments long enough to acquire 8,000 surface-reflected photons or up to a maximum length of 7 km. Minimum ocean segment lengths are usually 3 or 4 km. To better exclude subsurface returns under the crests of waves, surface finding is actually done on the basis of histograms of the photon height anomalies relative to a running 11-point average of the photon heights deemed high confidence surface photons in ATL03. The histogram of these anomalies is then trimmed of noise photons in the high and low tails of the distribution. Once the surface photons are so identified, their actual heights are used in subsequent processing.

To account for the instrumental uncertainty in photon time of flight due mainly to uncertainty in the start time of the photon flights within each laser pulse, the instrument impulse response histogram derived from the downlinked Transmit Echo Pulse (TEP) is deconvolved from the received height histogram to yield a surface histogram.

The ATL12 main outputs are the mean and next three moments of the resulting histogram. The 10-m along-track bin averages of photon heights are computed and used to determine electromagnetic (EM) sea state bias (hereafter referred to as SSB) and wave harmonics projected on to the ground track direction. Uncertainty in the mean surface height is largely due to sampling the wave covered surface and is proportional to significant wave height (SWH) and inversely proportional to the square root of ocean segment length divided by the correlation length scale. For ATL12 Release 4 and beyond, the 10-m along-track averages are used to yield the track-projected wave harmonics, correlation scale, and degrees of freedom.

In addition to data from ICESat-2 ATL03, ATL12 pulls in data from outside sources such as ocean depth from GEBCO and in Release 5, ice concentration from NSIDC.

1.2 ATL19 Gridded Product

The ATL19 gridded product is intended to provide users with a realization of the height of the ocean surface mapped over the world ocean in 1-month (and ultimately 3-month) averages. This contrasts with the ATL12 ocean surface height, which is an along-track record of sea surface height and related variables, each file of which covers only four ICESat-2 orbits representing 6
The primary ATL19 gridded product is dynamic ocean topography (DOT), which is sea surface height relative to the WGS84 ellipsoid minus the height of the EGM 2008, mean-tide geoid [Neumann et al., 2021b] relative to the WGS84 ellipsoid. The ATL12 processing mainly works with DOT to avoid the large variations associated with the geoid, but consistent with prior NASA planning ATL12 outputs ocean segment averages of the SSH and the geoid required to compute ocean segment-averages of DOT. We chose the primary output of ATL19 to be DOT (with the corresponding geoid as an ancillary variable to enable determination of SSH) because the variations in DOT represent familiar circulation patterns and because being much smaller than SSH variations, inter-beam biases and error stand out sharply in DOT.

1.2.1 ATL19 Grids

ATL19 uses 3 grids, North and South polar stereographic 25-km grids as well as an overlapping mid-latitude curvilinear ¼° latitude-longitude grid between 60°S and 60°N. The gridding is done individually for each beam on the ocean segments for each beam with average positions inside a grid cell.

1.2.2 The Basic Product

The basic product includes one-month simple averages and averages weighted by the estimated degrees of freedom for each beam ocean segment. Computing the individual beam averages provide a measure of relative biases among the six beams. The simple and degree-of-freedom weighted average grid or latitude-longitude positions of all the beam ocean segments in a grid cell are also output as are the simple and degree-of-freedom weighted averages of other key variables necessary to interpret DOT, such as the geoid and the sea state bias are also provided.

1.2.3 All-beam and running 3-month averages

The present release includes all-beam averages and planar fits over 9 cells to interpolate DOT to grid cell centers. ATL23 is based on three-month running averages of the ocean segment DOT which provides more complete filling of grid cells and better interpolation of DOT to the center of the grid cells. ATL23 grids are provided for each month using a sliding three-month window. The ATL23 file naming convention uses the first month in the file name, with ATL12 data from the two subsequent months forming each three-month ATL23.

1.2.4 Future Enhancement: Merging with ATL10 to produce global DOT

DOT as provided by ATL12/19/23 in ice covered oceans is biased by the freeboard of the sea ice. In later releases of ATL19/23 we will account for this in two phases. First, we will work to reconcile any biases between ATL12-derived DOT and DOT from the ATL10 sea ice freeboard product in the low ice concentration regions of the marginal ice zone (MIZ). One possibility that shows initial agreement is to compare ATL10 to the lower of the two ATL12 surface height distributions in the 2-Gaussian mixture representation provided by ATL12 of the DOT distribution. At low ice concentrations we expect this lower component of the Gaussian mixture
represents the sea surface and the higher component the ice surface. Once basic biases between ATL10 and ATL12 in the MIZ are resolved, we can subtract the ATL10 freeboard from the apparent ATL12 DOT to yield the true DOT in higher ice concentration regions. Work on this is ongoing with the ICESat-2 Project Office.

1.2.5 Future Enhancement: Optimal interpolation of DOT

One-month gridded and even three-month gridded ICESat-2 data have unfilled grid cells. We want to provide the ATL19/23 user with as much information as possible for ICESat-2 to be optimally interpolated over a wide a range of regions and temporal resolutions as well as optimally interpolated global maps of DOT in ATL19/23. We think the 3-month moving averages that will be part of ATL23 are candidates for the background fields (B in Appendix E) underlying DOT anomalies to be interpolated at finer scales. ATL12 and ATL19/23 are unique in providing degree-of-freedom and uncertainty estimates for ocean segment and gridded DOT, which provide measurement error values for each grid cell observation (D in Appendix E). In the future, the key added product in ATL19/23 will be maps of correlation length scales, possibly in two directions, based on the covariance of regional groups of our ¼° and polar stereographic gridded ICESat-2 DOT (R in Appendix E).
2.0 GRIDDED OCEAN PRODUCT (ATL19/23 L3B)

2.1 Gridded DOT

ATL19, based on ATL12, contains gridded monthly estimates of DOT from all ICESat-2 tracks from the beginning to the end of each month. ATL23, similarly, contains gridded estimates spanning three-months, from the beginning of the first month to the end of the third month. Below 60°N and above 60°S, the data are mapped on the ¼° curvilinear latitude-longitude grid. In response to reviewer comments, these latitude limits will be increased in the future to 66°N and 66°S to match the region of TOPEX/Poseidon coverage. Above 60°N and below 60°S, the grid data are mapped onto a planimetric grid using the NSIDC Sea Ice Polar Stereographic grids (https://nsidc.org/data/polar-stereo/ps_grids.html) with a grid spacing of 25 km. In the polar oceans the ATL10 sea ice products and ATL21 gridded sea ice products will eventually be reconciled with ATL12 and ATL19 data by methods TBD.

2.1.1 Grid Parameters

2.1.1.1 DOT

With only ATL12 needed as input, the primary ATL19/23 will be grid cell averages of product dynamic ocean topography (DOT), the sea surface departure from the EGM2008 mean-tide geoid. These include simple arithmetic 1-month averages of DOT, degree-of-freedom-weighted averages and multi-cell, least-squares linear interpolations to grid cell centers. In addition to the mean, the product will include standard deviation, skewness, and kurtosis, propagated from 2nd, 3rd, and 4th moments from ATL12 ocean segments included in each grid cell.

The corresponding averages of position, geoid height, SSB, ocean depth, ice concentration, and other pertinent parameters from each segment will also be output. The mean SSH can be calculated as the mean DOT plus the weighted average geoid height.

2.1.1.2 Sea surface statistics histogram within grid

For each month, the aggregate histogram of photon heights expressed as DOT accumulated in the cell for all ocean segments in a grid cell will be output. The mean SSH can be calculated as the mean DOT plus the weighted average geoid height.

2.1.1.3 Wave statistics within grid

Estimates of SWH and SSB from the \textit{a priori} estimation of sea state bias will also be grid-averaged with appropriate normalization for the number of surface photons in each segment.
3.0 ALGORITHM IMPLEMENTATION

This section provides a more detailed description of the calculations of the ATL19 gridded products. It is meant to guide the derivation of both the development MATLAB code and the NASA ASAS computer code that will be used to produce ATL19. During development of the ASAS code, its output will be checked against the MATLAB code for selected ATL12 input data.

3.1 Block Diagram for ATL19/23 Processing

This product, based on Product ATL12/3A, contains gridded monthly estimates of DOT from all ICESat-2 tracks from the beginning to the end of each month. Below 60°N and above 60°S, the data are mapped on the ¼° curvilinear latitude-longitude grid. In response to reviewer comments, these latitude limits will be increased in the future to 66°N and 66°S to match the region of TOPEX/Poseidon coverage. Above 60°N and below 60°S, the grid data are mapped onto a planimetric grid using the NSIDC Sea Ice Polar Stereographic grids (https://nsidc.org/data/polar-stereo/ps_grids.html) with a grid spacing of 25 km.

ATL12 provides the histograms and first four moments of dynamic ocean topography over ocean segments up to 7-km long (DOT can be converted to SSH by adding ocean segment average geoid height, which is also output by ATL12). It also provides the number of photon

![Block Diagram for ATL19/23 Processing](image)

Figure 2. Block diagram for the ATL19 gridding procedure taking ATL12 ocean products as input. \( \mu \), \( \sigma \), \( \Sigma \), and \( K \) denote mean, standard deviation, skewness, and kurtosis respectively.

- heights, \( n_{\text{photons}} \), going into the moments and an effective degrees-of-freedom, \( NP_{\text{effect}} \),

- the derivation of both the de
based on the correlation length scale of surface heights. Using these, ATL19/23 will produce monthly aggregate histograms of surface heights and averages of the ocean segment moments weighted by both \( n_{\text{photons}} \) and \( h_{\text{unrtn}} \) (Fig. 2).

### 3.2 Gridding DOT for ATL19/23

The ATL19/23 product includes gridded monthly estimates of dynamic ocean topography (DOT) taken from ATL12 ocean segment data. Ocean segments range in length roughly from 3 to a maximum 7 km dependent on photon rate. For ATL19/23 the ocean segment data are averaged in \( \frac{1}{4}\degree \) latitude-longitude or 25-km polar stereographic grid cells. Data from all six beams are used, both individually and averaged together from the beginning to the end of each month, prior to the summer of 2021 only strong beam data were downlinked and available over most of the ocean.

#### 3.2.1 The Grids

The ICESat-2 data from ATL12 are averaged onto three grids, called mid-latitude, north-polar and south-polar. The ATL19/23 data file has groups with similar names containing the gridded data from each of those regions. It is important to note that when we do gridding of individual beams (or in ATLAS terminology: spots) it does not imply that the individual ground tracks, gt1l; gt1r; gt2l; gt2r; gt3l; gt3r, from ATL12 are averaged. This is avoided due to the fact that the ground track of a beam changes depending on the flight direction of the spacecraft (Fig. 1) For ATL19/23, strong beams are kept together over yaw flips. This is so that knowing average DOT differences across grid cells is small, we can use the individual beam gridded DOT values for calculating the bias between spots/beams.

![Figure 3. Number of ocean segments found in each grid cell of the \( \frac{1}{4}\degree \) mid-latitude grid in August 2020.](image)

The mid-latitude group contains ocean segment data mapped onto the curvilinear, \( \frac{1}{4}\degree \) latitude-longitude grid extending from 60\degree S to 60\degree (Fig. 3), to be expanded in future releases to 66\degree S to 66\degree N. The grid cells are centered on the odd 1/8\textsuperscript{th} degree, with the latitude and longitude matrices defined in gridcntr_lat and gridcntr_lon, respectively. The matrix size of the gridded variables in the mid-latitude group is [480 x 1440].

As with the ATL20 product, ATL19/23 uses the North and South NSIDC Sea Ice Polar Stereographic grids (Fig. 4, https://nsidc.org/data/polar-stereo/ps_grids.html) to project data poleward of 60 degrees latitude. Both grids have a grid spacing of 25 km, equivalent to \( \frac{1}{4}\degree \) degree of latitude and are relative to the Hughes 1980
Ellipsoid. The origins of these grids are at the poles and expressed in x and y distances from the poles. The North polar grid ([https://epsg.io/3411](https://epsg.io/3411)) matrix is of size [448 x 304], with a true distance at 70°N and a central longitude along 45°W-135°E, with y positive along 135°E and x positive along 45°E. The South polar grid ([https://epsg.io/3412](https://epsg.io/3412)) matrix is of size [332 x 316], with a true distance at 70°S and a central longitude along 180°-0°E with y positive along 0°E and x positive along 90°E. The ATL19/23 defined grid variables for the polar regions are \textit{ds\_grid\_x} and \textit{ds\_grid\_y}. The north-polar and south-polar groups also contain \textit{gridcntr\_lat} and \textit{gridcntr\_lon}, similar to the mid-latitude group with latitude and longitude values converted from the x and y values in \textit{ds\_grid\_x} and \textit{ds\_grid\_y}.

Major portions of each of these grids are not ocean, and the gridded sea surface height values for these grid cells will be set to a default invalid value.

### 3.2.2 Temporal Averaging

ATL19 includes monthly one-month averages and ATL23 includes monthly, 3-month moving averages. The monthly 1-month data include aggregate histograms of DOT and averages of the ocean segment moments for data from all beams together and for each beam individually. The monthly gridded averages mid-latitude and polar grids do not produce averages for every grid cell, the sparseness of the averages being most pronounced at low latitudes. At the equator, the ICESat-2 orbits provide only one satellite pass per \(\frac{1}{4}\)° of longitude over the 91-day repeat cycle.
Consequently, to provide better data coverage and allow a least squares linear interpolation of DOT to grid cell centers, ATL23 includes a least-squares planar fit among 9 (3X3) grid cells to grid cell centers. The goal is to provide data for every grid cell that is not perpetually under a heavy cloud cover.

### 3.2.3 Input to Gridding

Input to the ATL19 gridding process for each beam (ATLAS spot) includes the first four moments of sea surface height, $h$, mean SSH, $h_{\text{var}}$, variance of SSH, $h_{\text{skewness}}$, and kurtosis of SSH, $h_{\text{kurtosis}}$, for each ocean segment from ATL12 (the moments in ATL12 are computed on DOT and the geoid is added to the mean to produce $h$). Simple averages and averages weighted by the degrees-of-freedom for each ocean segment are included. Prior to the February 8, 2022 version of this ATBD and in all ATL12, we have considered DOT strictly as surface height, $h$, minus the geoid. We have not applied a sea state bias correction, leaving that to the user. For the purposes of gridding, we have decided to apply the correction for sea state bias because for computing centered averages the coefficients for the planar fit of sea state bias would not necessarily be the same as those for the surface height minus the geoid. Appropriately weighted gridded sea state bias estimates will be included so that users can remove the sea state bias correction from gridded DOT. Thus, for the purpose of gridding, the gridded DOT is taken as sea surface height, $h$, minus the geoid height, $\text{geoid}_\text{seg}$, and minus bin_ssbias, i.e., $\text{DOT} = h - \text{geoid}_\text{seg} - \text{bin_ssbias}$. * The gridded DOT variance, skewness, and kurtosis are derived from $h_{\text{var}}$, $h_{\text{skewness}}$, and $h_{\text{kurtosis}}$ respectively. Other geophysical variables gridded from the ATL12 include significant wave height, $\text{swh}$, sea state bias, $\text{bin_ssbias}$ and the aggregation of $\text{y}$ histograms. (Note: ATL12 $\text{y}$ is the histogram of DOT over an ocean segment minus $\text{meanoffit2}$, which is a preliminary mean DOT over the ocean segment.) Sea surface height uncertainty, $h_{\text{uncrtn}}$, as well as the photon rate ($\text{photon\_rate}$) and the photon noise rate ($\text{photon\_noise\_rate}$) are also gridded. See Table 3 for complete list of ATL19 variables.

*Reminder: In ATL12 processing to reduce variability due to the considerable non-oceanographic variation in the geoid, we work with DOT, the anomaly of photon heights about the geoid. ATL12 outputs sea surface height, $h$, relative to the WGS84 ellipsoid to be consistent with other ICESat output. For ATL12 output, ocean segment DOT is converted to mean sea surface height, $h$, by adding ocean segment mean geoid height, $\text{geoid}_\text{seg}$, which is also output by ATL12.

For gridding purposes, ATL12 provides the number of photon heights, $n_{\text{photons}}$, in each ocean segment used to determine the DOT moments. It also provides the effective degrees-of-freedom, $np_{\text{effect}}$, which are based on the correlation length scale of surface heights and allow computing grid averages weighted by degrees of freedom. See Table 3 for complete list of ATL19 variables.

### 3.2.3.1 Pre-grid Filtering – Along-Track

The ATL12 ocean segment data going into ATL19/23 are already filtered for depths greater than 10-m and for pointing and orbit determinations outside of nominal conditions (ATL03 podppd flag = 0, nominal, or 4, nominal calibration maneuver). Ocean segment averages of depth, $\text{depth\_seg}$, and the highest of the podppd flag used in an ocean segment, $\text{podppd\_flag\_seg}$, are included in ATL12 output. ATL19 gridded averages of these quantities are computed as discussed below and in Table 3.
We find that for reasons that we are investigating, ATL12 produces some ocean segment heights that are unrealistic compared to the geoid. For each 4-orbit ATL12 file, data from all six beams are concatenated. Means of dynamic ocean topography (DOT equal to $h_{\text{geoid\_seg-bin\_ssbias}}$) are computed for each of the 18 ten-degree latitude bands for an ATL12 file. The latitude bands are centered on the $5^\circ$ marks and do not overlap, i.e. $\{90^\circ S \leq \text{ATL12 latitudes} < 80^\circ S\}$, $\{80^\circ S \leq \text{ATL12 latitudes} < 70^\circ S\}$, … $\{80^\circ N \leq \text{ATL12 latitudes} < 90^\circ N\}$). The standard deviation, $\sigma$, of the DOT from the entire ATL12 file is also computed, and DOTs that are outside of $\pm 3\sigma$ from the associated latitude-band mean, are removed.

Table 1 Inputs to Ocean Gridded Products from ATL12
(See Table 6 in ATL12 ATBD for all ATL12 Outputs)

<table>
<thead>
<tr>
<th>Product Label</th>
<th>Units</th>
<th>Description</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ds_y_bincenters$</td>
<td>meters</td>
<td>Bin centers for $y$ probability density function -15 to +15, by 1 cm bins</td>
<td></td>
</tr>
<tr>
<td>gtx/ssh_segments/</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>latitude</td>
<td>degrees</td>
<td>Mean latitude of surface photons in segment</td>
<td>$lat_seg$</td>
</tr>
<tr>
<td>longitude</td>
<td>degrees</td>
<td>Mean longitude of surface photons in segment</td>
<td>$lon_seg$</td>
</tr>
<tr>
<td>gtx/ssh_segments/hights</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>Meters</td>
<td>Mean sea surface height relative to the WGS84 ellipsoid</td>
<td>$SSH$</td>
</tr>
<tr>
<td>$h_var$</td>
<td>meters$^2$</td>
<td>Variance of best fit probability density function (meters$^2$)</td>
<td>$SSH_var$</td>
</tr>
<tr>
<td>$h_skewness$</td>
<td></td>
<td>Skewness of photon sea surface height histogram</td>
<td>$SSH_skew$</td>
</tr>
<tr>
<td>$h_kurtosis$</td>
<td></td>
<td><em>Excess kurtosis of sea surface height histogram</em></td>
<td>$SSH_kurt$</td>
</tr>
<tr>
<td>meanoffit2</td>
<td>Meters</td>
<td>Mean of linear fit removed from surface photon height expressed as DOT during surface finding</td>
<td>$meanoffit2$</td>
</tr>
<tr>
<td>$y$</td>
<td>$m^{-1}$</td>
<td>Probability density function of photon surface height</td>
<td>$Y$</td>
</tr>
<tr>
<td>length_seg</td>
<td>Meters</td>
<td>Length of segment (m)</td>
<td>$length_seg$</td>
</tr>
<tr>
<td>binsize</td>
<td>Meters</td>
<td>Bin size for $Y$ and sshx</td>
<td>$binsize$</td>
</tr>
<tr>
<td>bin_ssbias</td>
<td>Meters</td>
<td>Sea state bias estimated from the correlation of photon return rate with</td>
<td>$bin_SSbias$</td>
</tr>
</tbody>
</table>
### Gridding

The ATL19 gridding process involves three general steps: binning, averaging, and interpolation to grid cell center. There are two averaging methods; simple averaging, and averaging weighted...
by the number of degrees of freedom of the ocean segment data. Data interpolated to grid cell center are also included in the ATL19 data product.

3.2.4.1 Binning

Consider one month of ATL12 concatenated data for each beam. Using the latitude and longitude from ATL12, lat SEG, and lon SEG, find the data that fall within a grid cell. For the polar grids, first convert the latitude and longitude to the appropriate polar stereographic coordinates x SEG and y SEG using libraries located at:

https://nsidc.org/data/polar-stereo/tools_geo_pixel.html with coordinate transforms for lat, lon to x, y and x, y to lat, lon also given in Appendix D. The appropriate grid bin containing each ocean segment can then be identified based on x SEG and y SEG and the x and y boundaries of the grid cells. Once the correct bin is identified, the data corresponding to Table 1 for that ocean segment is accumulated. This will result in each grid cell having a collection of the data from all n SEG ocean segments contained in the grid cell for each beam (beam_1, beam_2, etc.).

Once the ocean segments appropriate to each bin are identified, compute n PH SRFC, the sum of the number of surface reflected photons, n PHOTONS, for all ocean segments in the grid cell. Also compute n PHS TTL as the grid cell-total of all photons in the downlink bands, n TTL PHOTON. Compute dof equal to the sum of all the NP EFFECT in the bin. Compute the total length of all ocean segments in the bin, length SUM, as the sum of length SEG. Also output the number of segments in the bin, n SEGs. Additionally, compute the grid cell-aggregate photon rate, r SRFC, equal to n PH SFC divided by the total length of segments in the bin, length SUM. Finally, compute the grid cell noise rate, r NOISE, equal to (n PHS TTL minus n PH SRFC) divided by length SUM.

3.2.4.2 Individual Beam Averaging

3.2.4.2.1 Averaging over n SEGs Segments

For each grid cell and each beam with the accumulated data of n SEGs ocean segments compute outputs:

dot AVG, lat AVG, lon AVG, ssh AVG, geoid AVG, depth AVG, ice conc, and surf prcnt AVG as simple averages of:

SSH- geoid SEG-bin ssh bias, lat SEG, lon SEG, bin ssh bias, geoid SEG, depth ocn SEG, ice conc, and surf type prcnt.

(In a related calculation, length SUM will be computed as the sum of single beam ocean segment lengths, length SEG, and length SUM ALBM will be computed as the sum of all beam ocean segment lengths.)

Simple average is defined by the sum of the n SEGs ocean segment values of these variables divided by n SEGs. See Figure 5 for the August 2020 strong Beam-1 (Fig. 5 left) and strong Beam-3 (Fig. 5 right) “n SEGs” averages, from our Matlab developmental code.

To compute the bin average standard deviation, dot sigma AVG, of DOT variability over ocean segments, sum SSHVAR, divide by n SEGs, and take the square root to establish the average
standard deviation. Note that this and the other average moments do not include the ocean-segment-to-ocean-segment variability within the cell. This is likely much smaller than the variability due to sea state, but in future releases we plan to distinguish the ocean-segment-to-ocean-segment variability where adequate ocean segments are included by a TBD method.

Similarly, to compute the bin average significant wave height, \( SWH_{\text{avg}} \), sum \( (SWH)^2 \), divide by \( n_{\text{segs}} \), and take the square root to establish the average significant wave height.

To compute the bin average skewness, \( \text{dot}_{\text{skew}}_{\text{avg}} \), of DOT, sum \( SSH_{\text{skew}} \times (SSH_{\text{var}})^{1/2} \), divide by \( n_{\text{segs}} \), and divide by \( \text{dot}_{\text{sigma}}_{\text{avg}}^3 \) to establish the average skewness.

To compute the bin average excess kurtosis, \( \text{dot}_{\text{kurt}}_{\text{avg}} \), of DOT, sum \( (SSH_{\text{kurt}} + 3) \times (SSH_{\text{var}})^2 \), divide by \( n_{\text{segs}} \), and divide by \( \text{dot}_{\text{sigma}}_{\text{avg}}^4 \). Subtract 3 to establish the average excess kurtosis.

To compute the uncertainty, \( \text{dot}_{\text{avg}}_{\text{uncrtn}} \), in gridded DOT, \( \text{dot}_{\text{avg}} \), divide \( \text{dot}_{\text{sigma}}_{\text{avg}} \) by the square root of \( \text{dof} \) to establish the uncertainty in the degree-of-freedom weighted DOT. As with the higher moments, this is the uncertainty due to sea state and does not include the ocean-segment-to-ocean-segment variability within the cell. In future releases we plan to estimate the ocean-segment-to-ocean-segment uncertainty where adequate ocean segments are included by a TBD method.

To compute the bin aggregate probability density function (PDF), \( \text{dot}_{\text{hist}} \), of DOT, we first must convert each \( Y \) PDF from ATL12 to a PDF of DOT by adding \( \text{meanoffit2} \) to the x-axis of \( Y, ds_y_{\text{bincenters}} \) and then interpolating the result to an intermediate PDF, \( Y_{\text{intermediate}} \), evaluated at the original \( ds_y_{\text{bincenters}} \). (Note: In ATL19 Release 1, meanoffit2 was inadvertently not added so the aggregate histograms only reflect the aggregate wave environment with mean near zero.) The aggregate probability PDF, \( \text{dot}_{\text{hist}} \), of DOT will equal the sum.
Intermediate $n_{\text{photons}}$ in each histogram bin of all Intermediate divided by the total, $n_{\text{photons}}_{\text{gridttl}}$, of all $n_{\text{photons}}$.

Note that in the implementation of Release 2 of ATL19, we found that including $\text{dot_hist}$ for each individual, beam each grid cell, and 6001 histogram bins used an excessive amount of storage necessitating the computation of $\text{dot_hist}$ only as an aggregate for all beams termed $\text{dot_hist}albm$.

### Table 2: Mid-Latitude Inter-Beam Biases, Oct. & Nov. 2020

<table>
<thead>
<tr>
<th>No Ocean Scans</th>
<th>Beam 1</th>
<th>Beam 2</th>
<th>Beam 3</th>
<th>Beam 4</th>
<th>Beam 5</th>
<th>Beam 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam 1</td>
<td>0.0066</td>
<td>0.0066</td>
<td>0.0021</td>
<td>0.0066</td>
<td>-0.0089</td>
<td></td>
</tr>
<tr>
<td>Beam 2</td>
<td>-0.0066</td>
<td>-0.0024</td>
<td>-0.0017</td>
<td>0.0024</td>
<td>-0.0107</td>
<td></td>
</tr>
<tr>
<td>Beam 3</td>
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<td>0.0024</td>
<td>-0.0001</td>
<td>-0.0001</td>
<td>-0.0116</td>
<td></td>
</tr>
<tr>
<td>Beam 4</td>
<td>-0.0021</td>
<td>0.0017</td>
<td>0.0001</td>
<td>0.0041</td>
<td>-0.0096</td>
<td></td>
</tr>
<tr>
<td>Beam 5</td>
<td>-0.0066</td>
<td>-0.0024</td>
<td>0.0001</td>
<td>-0.0041</td>
<td>-0.0152</td>
<td></td>
</tr>
<tr>
<td>Beam 6</td>
<td>0.0089</td>
<td>0.0107</td>
<td>0.0116</td>
<td>0.0096</td>
<td>0.0152</td>
<td></td>
</tr>
</tbody>
</table>

#### 3.2.4.2.2 Averaging Weighted by Degrees-of-Freedom

To account for the different uncertainties in linear variables (e.g., DOT) the averaging is as in 3.2.4.2.1 except the variables are weighted by the effective degrees of freedom, $NP_{\text{effect}}$, of each ocean segment DOT. These degree-of-freedom weighted averages may be different from simple averages in important ways for cases where different beams in a cell measure over different sea states and have different sea state induced DOT uncertainty. DOT measured under calm conditions will be more certain than DOT measured over a rough sea surface.

For each grid cell and each beam with the accumulated data of $n_{\text{segs}}$ ocean segments compute outputs:

\[ \text{dot\_dfw, lat\_dfw, lon\_dfw, ssb\_dfw, geoid\_dfw, depth\_dfw and length\_dfw} \]

as degree-of-freedom weighted averages of:

\[ \text{SSH\_geoid\_seg-bin\_ssbias, lat\_seg, lon\_seg, bin\_ssbias, geoid\_seg, depth\_ocn\_seg, and length\_seg}. \]

Degree-of-freedom averages are found by first taking the sum of the $n_{\text{segs}}$ ocean segment values multiplied by their respective ocean segment $NP_{\text{effect}}$ and then dividing by $dof$, which
is equal to the sum of all the $NP_{\text{effect}}$ in the bin to establish the degree-of-freedom weighted averages.

To compute the bin degree-of-freedom weighted average standard deviation of DOT, $\text{dot}_{\text{sigma}}_{\text{dfw}}$, of DOT, sum $SSH_{\text{var}}$ multiplied by $NP_{\text{effect}}$. Then divide by $dof$ and take the square root to establish the degree-of-freedom weighted standard deviation of DOT. Note that this and the other average moments do not include the ocean-segment-to-ocean-segment variability within the cell. This is likely much smaller than the variability due to sea state, but in future releases we plan to distinguish the ocean-segment-to-ocean-segment variability where adequate ocean segments are included by a TBD method.

Similarly, to compute the bin degree-of-freedom weighted average significant wave height, $\text{SWH}_{\text{dfw}}$, sum $(\text{SWH})^2$ multiplied by $NP_{\text{effect}}$. Then divide by $dof$ and take the square root to establish the degree-of-freedom weighted average significant wave height.

To compute the bin degree-of-freedom weighted average skewness, $\text{dot}_{\text{skew}}_{\text{dfw}}$, of DOT, sum $SSH_{\text{skew}} \times (SSH_{\text{var}})^{1/2}$ multiplied by $NP_{\text{effect}}$. Then divide by $dof$ and divide again by $\text{dot}_{\text{sigma}}_{\text{dfw}}^3$ to establish the degree-of-freedom weighted skewness.

To compute the bin degree-of-freedom weighted average excess kurtosis, $\text{dot}_{\text{kurt}}_{\text{dfw}}$, of DOT, sum $(SSH_{\text{kurt}}+3) \times (SSH_{\text{var}})^2$ multiplied by $NP_{\text{effect}}$. Then divide by $dof$ and divide again by $\text{dot}_{\text{sigma}}_{\text{dfw}}^4$. Subtract 3 to establish the degree-of-freedom weighted excess kurtosis.

To compute the uncertainty in gridded DOT, $\text{dot}_{\text{dfw}}$, divide $\text{dot}_{\text{sigma}}_{\text{dfw}}$ by the square root of $dof$ to establish the uncertainty in the degree-of-freedom weighted DOT. As with the higher moments, this is the uncertainty due to sea state and does not include the ocean-segment-to-ocean-segment variability within the cell. In future releases we estimate the ocean-segment-to-ocean-segment uncertainty where adequate ocean segments are included using a least-squares weighted by a function of $h_{\text{uncrt}}$.

### 3.2.4.2.3 Inter-Beam Biases

The procedures of section 3.2.4.2 will be performed independently for each beam, if for no other reason than a particular satellite pass may have ground tracks in adjacent pairs of cells. Furthermore, comparing the gridded product for the individual beams will disclose instrumental biases. For example, for August 2020, biases between the strong beams were significantly less than a centimeter (Figure 5), and Table 2 shows that the mid-latitude grid average inter-beam biases for October-November 2020 were mostly less than a centimeter. In the future inter-beam biases can be monitored with gridded single-beam averages and accounted for by a TBD method in a gridded product that combines all the beams.
3.2.4.3 All-beam Averages

We want all-beam quantities for each grid cell to achieve grid cell averages with maximum degrees of freedom and minimum uncertainty. Figure 6 shows DOT gridded by our developmental code in the mid-latitude grid by simple averaging (left) and degree-of-freedom weighted averaging (right).

Figure 6. Mid-latitude DOT gridded by simple “n-segment” all-beam averages (left) and degree-of-freedom weighted (dfw) all-beam averages (right) for August 2020. The ocean scan region in the Central Pacific is not gridded.

All-beam quantities will mirror the single beam totals and averages of section 3.2.4.2 and use the same names with the suffix “_albm” appended (See Table 3). The all-beam variables are computed in the same way the single beam variables are computed except all the ocean segments from all beams in a grid cell are used in the computation. These values should be the same as appropriately weighted averages of grid cell single beam averages (See Appendix D).

3.2.4.4 Interpolation of DOT to Bin Centers

To compute the average DOT at the center of each grid cell, we perform least squares fit of a plane of the form \( dot = a \times x + b \times y + c \) to the ocean segment DOT in each cell plus the eight surrounding cells and evaluate the fitted plane at the center of the center cell. For a sensible solution we require data from at least two ICESat-2 orbits. It is due to the spacing of the orbits we look for data in nine cells; the center cell where we will compute the center-interpolated value and the eight cells surrounding that center cell. There must also be a minimum of four ocean segments in the 9 cells to compute a center-interpolated DOT value. Because we need as much data as possible to compute center values, all center values denoted by the suffix “\texttt{cntr}” will be derived using all beams. Further averages over the center cell and eight surrounding cells will be denoted by the suffix “\texttt{9}” and will be all-beam averages.

Note that in interpolating centered averages it is possible that center points can be on land, e.g., an island or peninsula. A land mask is provided as output to indicate if a center point for a centered average is in the ocean (\texttt{landmask}=1) or on land (\texttt{landmask}=0). While one would
not likely use the centered DOT value itself, the associated \( a, b, \) and \( c \) planar coefficients are from ocean data only, can be used to interpolate DOT in the water region surrounding the land, and are thus be of particular benefit to coastal oceanography and as a basis for optimal interpolation.

### 3.2.4.1 Average DOT at Bin Centers

For the simple averages: Computing averages interpolated to grid cell centers requires data from at least two orbits to get the required horizontal distribution of data to make a meaningful least-squares fit in space. The ICESat-2 orbital characteristics require that for a 1-month average at a grid cell center we must consider data in the surrounding 8 cells for a 9–cell fit.

For \( n_{\text{segs}} \) greater than or equal to 4 from at least two orbits, assemble 1 by \( n_{\text{segs}} \) vectors of the deviation of the DOT values from their average values, \( \text{dot}_{\text{avg}}9, \text{lon}_{\text{avg}}9, \) and \( \text{lat}_{\text{avg}}9 \), where we are including the data from all nine cells for those averages.

\[
h_i' = (\text{SSH-geoid}_{\text{seg}}\text{-bin}_{\text{SSbias}})\text{-dot}_{\text{avg}}9 \quad i = 1 \text{ to } n_{\text{segs}} \tag{45}
\]

where \( \text{dot}_{\text{avg}}9 \) equals the 9-cell all-beam average of \( (\text{SSH-geoid}_{\text{seg}}\text{-bin}_{\text{SSbias}}) \)

\[
x_i' = \text{lon}_{\text{seg}}\text{-lon}_{\text{avg}}9 \quad i = 1 \text{ to } n_{\text{segs}} \tag{46}
\]

\[
y_i' = \text{lat}_{\text{seg}}\text{-lat}_{\text{avg}}9 \quad i = 1 \text{ to } n_{\text{segs}} \tag{47}
\]

Referring to Appendix B, compute the cross product expected values \( L_{xx}, L_{yy}, L_{xy}, R_{xh}, \) and \( R_{yh} \):

\[
L_{xx} = \sum_{i=1}^{N} x_i'x_i' \\
L_{yy} = \sum_{i=1}^{N} y_i'y_i' \\
L_{xy} = \sum_{i=1}^{N} y_i'x_i'
\]

\[
R_{xh} = \sum_{i=1}^{N} x_i'h_i' \\
R_{yh} = \sum_{i=1}^{N} y_i'h_i'
\tag{48}
\]

for \( N \) equal to \( n_{\text{segs}} \).

The coefficients defining the least-squares planar fit \( a, b, \) and \( c \) are given by

\[
a = \frac{R_{yh}L_{xy} - R_{xh}L_{yy}}{L_{xx}L_{yy} - L_{xy}^2} \\
b = \frac{R_{yh}L_{xx} - R_{xh}L_{xy}}{L_{xx}L_{yy} - L_{xy}^2} \\
c = \text{dot}_{\text{avg}}9 - (a*\text{lon}_{\text{avg}}9 + b*\text{lat}_{\text{avg}}9)
\tag{49}
\]

The values of the planar fit coefficients, \( a, b, \) and \( c \) should be relabeled, saved, and output as \( a\_avg, b\_avg, \) and \( c\_avg \) because they can be used as the linear model of DOT in the grid cell.
Together with the planar fits of all the other grid cells, they constitute a faceted linear model of DOT potentially over the whole world ocean that can form the basis model for optimal interpolation. The value of DOT at the center of the grid cell, \( \text{dot\_avgcntr} \) is then given by:

\[
\text{dot\_avgcntr} = a_{\text{avg}} \times \text{gridcntr\_lon} + b_{\text{avg}} \times \text{gridcntr\_lat} + c_{\text{avg}}
\]  

(50)

To calculate the effective sea state bias, \( \text{ssb\_avgcntr} \), we have to redo the calculation of the center value of DOT, \( \text{DOT\_avgcntr\_nobias} \), without the SSB correction using equation (45b) in place of (45):

\[
h_i = (\text{SSH\_geoid\_seg}) - \text{dot\_avgnossb9}, i = 1 \text{ to } n\_\text{segs}
\]  

(45b)

where \( \text{dot\_avgnossb9} \) equals the 9-cell all-beam average of \( (\text{SSH\_geoid\_seg}) \)

Then the effective SSB at the cell center will be \( \text{ssb\_avgcntr} = \text{DOT\_avgcntr\_nobias} - \text{dot\_avgcntr} \).

Calculate the uncertainty in the center average of DOT, \( \delta\hat{h} = \text{dot\_avgcntr\_uncrtn} \) in the planar fit estimate of DOT at the center position, \( \hat{x} = \text{gridcntr\_lon} \), and \( \hat{y} = \text{gridcntr\_lat} \), using equation (B14) of Appendix B for the uncertainty squared:

\[
(\delta\hat{h})^2 = (\delta\hat{h}_0)^2 \left( \frac{1}{L_a} \right)^2 \left[ (L_{xy} L_{xx} - L_{xy} L_{yy}) (\hat{x}')^2 + (L_{xy} L_{yy} - L_{xx} L_{xy}) (\hat{y}')^2 + 1 \right]
\]

(B14)

\[
\left( \frac{1}{L_a} \right)^2 = \frac{1}{(L_{xx} L_{yy} - L_{xy} L_{xy})^2}
\]

Where \( \hat{x}' = \hat{x} - \bar{x} \) and \( \hat{y}' = \hat{y} - \bar{y} \) and the mean data location, \( \bar{x} , \bar{y} \), is \( \text{lon\_avg} , \text{lat\_avg} \).

\( \delta\hat{h}_0 \) is the uncertainty in height relative to the quality of the planar fit (QoF) computed from the sum of the squared error about the fit according to (B8) or in the absolute coordinate system:

\[
\text{QoF} = \left( (N - 2)^{-1} \sum_{i=1}^{N} \left( \text{DOT\_seg} - (a_{avg} \times \text{lon\_seg} + b_{avg} \times \text{lat\_seg} + c_{avg}) \right)^2 \right)^{\frac{1}{2}}
\]

or

\[
\text{QoF} = (n\_\text{seg}/(n\_\text{seg}-2))^{1/2} \times \text{RMS} \left( \text{DOT\_seg} - (a_{avg} \times \text{lon\_seg} + b_{avg} \times \text{lat\_seg} + c_{avg}) \right)
\]  

(51)
To understand the effect of data spatial distribution on uncertainty, note that if the data point locations are randomly distributed, the $x$ and $y$ data positions are uncorrelated, $L_{xy}$ is zero and (B14) becomes:

$$\delta \hat{h}^2 = \delta \hat{h}_0^2 \left( \frac{1}{L_{xx} L_{yy}} \right)^2 \left( \left( L_{yy}^2 L_{xx} \right) \left( \hat{x}' \right)^2 + \left( L_{yy} L_{xx} \right) \left( \hat{y}' \right)^2 + 1 \right)$$

and the uncertainty increases away from the average data position as the ratio of extrapolation distance, $\hat{x}'$, to the RMS spread of the data in $x$, $L_{xx}$ and the ratio of extrapolation distance, $\hat{y}'$, to the RMS spread of the data in $y$, $L_{yy}$.

Conversely, if the $x$ and $y$ positions of the data are highly correlated, for example because they are on a single straight line, the correlation coefficient, $L_{xy}$ approaches zero, and the uncertainty becomes very large, especially for center positions well to the side of the correlated cloud of data points.

Therefore, we edit the centered average values with excessive uncertainty, $\text{dot\_avgcntr\_uncrtn} > a$ value estimated from examination of distributions of $\text{dot\_avgcntr\_uncrtn}$ to exceed more than 95% of $\text{dot\_avgcntr\_uncrtn}$ values. We expect this value to be around 0.2 m.

$\text{DOT\_seg} =$ ocean segment DOT data within the 9 cells and all beams  
$\text{lat\_seg} =$ latitudes of the ocean segments in the 9 cells and all beams for the mid-latitude grid.  
For the polar grids substitute the y-coordinate, $y\_seg$.  
$\text{lon\_seg} =$longitudes of the ocean segments in the 9 cells and all beams for the mid latitude grid.  
For the polar grids substitute the x-coordinate, $x\_seg$.

### 3.2.4.4.2 Degree-of-Freedom Weighted DOT at Bin Centers

*For the degree of freedom weighted averages:* To compute the average of degree-of-freedom weighted DOT at the center of each grid cell, start by assembling 1 by $n\_segs$ vectors of the deviation of the DOT values from their degree-of-freedom weighted average values, $\text{dot\_dfw9}$, $\text{lon\_dfw9}$, and $\text{lat\_dfw9}$, again using all-beam data from 9 (3x3) cells.

$$h_i = (NP\_effect)^{1/2} \ast (SSH - \text{geoid\_seg-bin\_SSbias- dot\_dfw9}), \quad i = 1 \text{ to } n\_segs$$
where $\text{dot}_\text{dfw9}$ equals the 9-cell all-beam degree-of-freedom weighted average of $(\text{SSH-geoid}_\text{seg-bin}_{\text{SSbias}})$

$$
\begin{align*}
  x_i' &= (\text{NP_effect})^{1/2} \times (\text{lon}_\text{seg} - \text{lon}_\text{dfw9}), \quad i=1 \text{ to } n_\text{segs} \\
  y_i' &= (\text{NP_effect})^{1/2} \times (\text{lat}_\text{seg} - \text{lat}_\text{dfw9}), \quad i=1 \text{ to } n_\text{segs}
\end{align*}
$$

(53)

(54)

We use $(\text{NP_effect})^{1/2}$ instead of $\text{NP_effect}$ above because in the next cross product calculation the multiplication is to amplify the sums by factors of $\text{NP_effect}$ as if the number of DOT values and their locations were expanded to number $\text{NP_effect}$. Referring to Appendix B, compute the cross product expected value $L_{xx}$, $L_{yy}$, $L_{xy}$, $R_{xh}$, and $R_{yh}$ using equations (B5).

The coefficients defining the least-squares planar fit ($a$, $b$, and $c$) are given by (B6). Relabel, save, and output the coefficients so that $a_\text{dfw}=a$, $b_\text{dfw}=b$, and $c_\text{dfw}=c$. The degree-of-freedom weighted value of DOT interpolated to the center of the grid cell, $\text{dot}_\text{dfwcntr}$ is then given by:

$$
\text{dot}_\text{dfwcntr} = a_\text{dfw} \times \text{gridcntr}\_\text{lon} + b_\text{dfw} \times \text{gridcntr}\_\text{lat} + c_\text{dfw}
$$

(54)

To calculate the effective sea state bias, $\text{ssb}_\text{dfwcntr}$, we must redo the calculation of the center value of DOT, $\text{DOT}_\text{dfwcntrnobias}$, without the SSB correction using equation (52b) in place of (52):

$$
\begin{align*}
  h_i' &= (\text{NP_effect})^{1/2} \times (\text{SSH-geoid}_\text{seg} - \text{dot}_\text{dfw9})_\text{nobias}, \quad i=1 \text{ to } n_\text{segs}
\end{align*}
$$

(52b)

where $\text{dot}_\text{dfw9}$ equals the 9-cell all-beam degree-of-freedom weighted average of $(\text{SSH-geoid}_\text{seg})$.

Then the effective SSB at the cell center will be $\text{ssb}_\text{dfwcntr} = \text{DOT}_\text{dfwcntrnobias} - \text{dot}_\text{dfwcntr}$.

We edit the degree-of-freedom weighted centered values (dot_dfwcntr and ssb_dfwcntr) with excessive uncertainty from the dot simple averaged center value uncertainty, $\text{dot}\_\text{avgcntr}\_\text{uncrtn}$. When $\text{dot}\_\text{avgcntr}\_\text{uncrtn} > 0.2 \text{ m}$ we edit the degree-of-freedom centered values as with the simple averaged centered values.
3.2.4.4.3 One-month ATL19 and Three-month ATL23 Centered Averages

Figure 7 shows DOT grid averages interpolated to cell centers using Equations (45)-(50) for 1-month ATL19, August 2020, (Figure 7, left) and 3-months ATL23, Jul.-Aug.-Sept. 2020, (Figure 7, right). The results are similar for degree-of-freedom averaging Section 3.2.4.4.2, Equations (51)-(54). Even at 1-month, the spatial averaging of the 9-cell fit results in significantly fewer empty cells than the simple averages (Figure 6, left). Furthermore, the 91-day repeat of ICESat-2 including 1397 orbits, with two equator crossings per orbit each, results in an equator crossing every 0.13 degrees of longitude, so that every ¼° grid cell has the potential, barring clouds, to see at least one satellite pass in 3 months, Consequently, almost every grid cell is filled in the 3-month centered average for July-Sept. 2020 (Figure 7, right). The 3-month centered average (Fig. 7, right) and degree-of-freedom weighted average will be good candidate background fields for optimal interpolation to finer spatial and temporal scales (Appendix E).

3.2.5 Gridding Output

Output of the ATL19 gridding process will come in three latitude groups: mid-latitude, north-polar, and south-polar for the three grid systems (Section 3.2.1). A hierarchy of the ATL12 and ATL19 variables is given in Appendix D. The generic list of output variables applicable to each latitude group is given in Table 3. The grid sizes vary with region: mid-latitude [480 x 1440] (y-index, x-index), north-polar [448 x 304] and south-polar [332 x 316].

Figure 7. Centered grid averages using 9-cells (3x3) to fit to the center of a center cell for 1-month ATL19, August 2020, (left) and 3-month ATL23, Jul-Aug-Sept. 2020, (right).
Table 3: ATL19 Outputs per Latitude Group

As indicated by "_albm", albm versions of the variables are also included

* Indicates in single beam groups only

<table>
<thead>
<tr>
<th>ATL19 Variable Name</th>
<th>Dimensions</th>
<th>Units</th>
<th>Description</th>
<th>Input ATL12 Variable Name</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a_avg</strong></td>
<td>mid-lat, north-polar, south-polar</td>
<td>m/deg m/m m/m</td>
<td>The $a$ coefficient of the planar fit used to compute dot_avgcntr values</td>
<td></td>
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<tr>
<td><strong>b_avg</strong></td>
<td>480 x 1440 448 x 304 332 x 316 INVALID_R8 B</td>
<td>m/deg m/m m/m</td>
<td>The $b$ coefficient of the planar fit used to compute dot_avgcntr values</td>
<td></td>
</tr>
<tr>
<td><strong>c_avg</strong></td>
<td>480 x 1440 448 x 304 332 x 316 INVALID_R8 B</td>
<td>m m m</td>
<td>The $c$ coefficient of the planar fit used to compute dot_avgcntr values</td>
<td></td>
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<tr>
<td><strong>a_dfw</strong></td>
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<td>m/deg m/m m/m</td>
<td>The $a$ coefficient of the planar fit used to compute dot_dfwcntr values</td>
<td></td>
</tr>
<tr>
<td><strong>b_dfw</strong></td>
<td>480 x 1440 448 x 304 332 x 316 INVALID_R8 B</td>
<td>m/deg m/m m/m</td>
<td>The $b$ coefficient of the planar fit used to compute dot_dfwcntr values</td>
<td></td>
</tr>
<tr>
<td><strong>c_dfw</strong></td>
<td>480 x 1440 448 x 304 332 x 316 INVALID_R8 B</td>
<td>m m m</td>
<td>The $c$ coefficient of the planar fit used to compute dot_dfwcntr values</td>
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<td><strong>delta_time</strong></td>
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<tr>
<td><strong>delta_time_end</strong></td>
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<td>seconds</td>
<td>Latest time in grid</td>
<td><strong>delta_time</strong></td>
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<td><strong>depth_ocn_seg</strong></td>
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<tr>
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<td>Simple average of ocean depth</td>
<td><strong>depth_ocn_seg</strong></td>
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</table>
ICESat-2 Algorithm Theoretical Basis Document for Gridded Dynamic Ocean Topography

Release 003(ATL19)/Release 001(ATL23)

<table>
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<th>Variable</th>
<th>Dimensions</th>
<th>Units</th>
<th>Description</th>
<th>Unit</th>
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<td>meters</td>
<td>Degree-of-freedom weighted average of ocean depth</td>
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<td>480 x 1440</td>
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<td>meters</td>
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<td>h-geoid_seg-bin_ssbias</td>
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<tr>
<td>dot_avgcenctr_uncrtin</td>
<td>480 x 1440</td>
<td>meters</td>
<td>Uncertainty of simple all-beam average of dynamic ocean topography interpolated to center of grid cell</td>
<td>h-geoid_seg-bin_ssbias</td>
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<td>h-geoid_seg-bin_ssbias</td>
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<td>dot_dfwcntr</td>
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<td>meters</td>
<td>Degree of freedom weighted all-beam average dynamic ocean topography</td>
<td>h-geoid_seg-bin_ssbias</td>
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<td>480 x 1440</td>
<td>meters</td>
<td>Uncertainty of degree of freedom weighted all-beam average of dynamic ocean</td>
<td>h-geoid_seg-bin_ssbias, NP_effect</td>
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<td>332 x 316</td>
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### Dot History and Beam Aggregate

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<th>Units</th>
<th>Notes</th>
</tr>
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<tbody>
<tr>
<td>dot_hist_albm</td>
<td>3001 x 480 x 1440</td>
<td>counts</td>
<td>All beam aggregate probability density function of DOT histograms (ATL12 Y histogram + measnoffit2). Histogram bin centers are given by ds_hist_bincenters for each of the three grids.</td>
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</tbody>
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### Dot Kurtosis and Skewness

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
<th>Notes</th>
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<tr>
<td>dot_kurt_avg</td>
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<td>none</td>
<td>Simple average of excess kurtosis of the dynamic ocean topography</td>
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<td>dot_kurt_dfw</td>
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<td>Degree of freedom weighted average of excess kurtosis of the dynamic ocean topography</td>
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<td>dot_skew_avg</td>
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<td>dot_skew_dfw</td>
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<td>Degree of freedom weighted average of the skewness of dynamic ocean topography</td>
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### Center X of Grid Cell

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<th>Units</th>
<th>Notes</th>
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<tbody>
<tr>
<td>ds_grid_x</td>
<td>304 (N) 316 (S)</td>
<td>meters</td>
<td>Center x value of polar grid cell</td>
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*ICESat-2 Algorithm Theoretical Basis Document for Gridded Dynamic Ocean Topography*  
*Release 003(ATL19)/Release 001(ATL23)*
<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Unit</th>
<th>Description</th>
<th>Definition</th>
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</thead>
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<tr>
<td>ds_grid_y</td>
<td>448 (N) 322 (S)</td>
<td>meters</td>
<td>Center y value of polar grid cell</td>
<td>defined</td>
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<tr>
<td>ds_hist_bincenters</td>
<td>3001</td>
<td>meters</td>
<td>Bin centers for DOT aggregate histograms, <em>dot_hist</em>, from -15m to +15m in 1-cm bins.</td>
<td><em>ds_y_bincenters</em></td>
</tr>
<tr>
<td>geoid_avg &quot;_&quot;_albm</td>
<td>480 x 1440 448 x 304 322 x 316 INVALID_R8 B</td>
<td>meters</td>
<td>Simple average of geoid height</td>
<td><em>geoid_seg</em></td>
</tr>
<tr>
<td>geoid_dfw &quot;_&quot;_albm</td>
<td>480 x 1440 448 x 304 322 x 316 INVALID_R8 B</td>
<td>meters</td>
<td>Degree of freedom weighted average of geoid height</td>
<td><em>geoid_seg</em></td>
</tr>
<tr>
<td>gridcntr_lat</td>
<td>480 x 1440 448 x 304 322 x 316</td>
<td>°N</td>
<td>Latitude of grid cell center</td>
<td>defined</td>
</tr>
<tr>
<td>gridcntr_lon</td>
<td>480 x 1440 448 x 304 322 x 316</td>
<td>°E</td>
<td>Longitude of grid cell center</td>
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<tr>
<td>ice_conc &quot;_&quot;_albm</td>
<td>480 x 1440 448 x 304 322 x 316 INVALID_R8 B</td>
<td></td>
<td>Simple average of ocean segment average ice concentration, <em>gtx/ssh_segments/stats/ice_conc</em> of ATL12</td>
<td><em>ice_conc</em></td>
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<tr>
<td>landmask</td>
<td>480 x 1440 448 x 304 322 x 316</td>
<td>none</td>
<td>A land mask to indicate if a center point for a 9-cell centered average is on land (1= ocean, 0=land), for example an island. While one would not use the centered DOT value, the <em>a</em>, <em>b</em>, and <em>c</em> coefficients are from ocean data only and can be used to interpolate DOT in the water region of the grid cell.</td>
<td></td>
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<td>latitude</td>
<td>480</td>
<td>°N</td>
<td>Vector of grid center latitude common values for all mid-latitude grid cells</td>
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<td>lat_avg &quot;_&quot;_albm</td>
<td>480 x 1440 448 x 304 322 x 316</td>
<td>°N</td>
<td>Simple average of latitude</td>
<td><em>latitude</em></td>
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<td>Variable</td>
<td>Dimensions</td>
<td>Units</td>
<td>Description</td>
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<td>°N</td>
<td>Degree of freedom weighted average of latitude</td>
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<tr>
<td></td>
<td>448 x 304</td>
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<td>332 x 316</td>
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<td></td>
<td>INVALID_R8 B</td>
<td></td>
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<tr>
<td><strong>length</strong> dfw _“_albm</td>
<td>480 x 1440</td>
<td>meters</td>
<td>Degree of freedom weighted average of segment length</td>
<td>length</td>
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<tr>
<td></td>
<td>448 x 304</td>
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<td>332 x 316</td>
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<td>INVALID_R8 B</td>
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<td><strong>length</strong> sum _“_albm</td>
<td>480 x 1440</td>
<td>meters</td>
<td>Sum of ocean segment lengths</td>
<td>length</td>
</tr>
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<td>448 x 304</td>
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<td><strong>n_ph</strong> sr fc _“_albm</td>
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<td>counts</td>
<td>Sum of surface reflected photons plus rejected photons</td>
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<td><strong>n</strong> segs _“_albm</td>
<td>480 x 1440</td>
<td>counts</td>
<td>Number of ocean segments in grid cell</td>
<td>n_segs</td>
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<td><strong>Units</strong></td>
<td><strong>Description</strong></td>
<td><strong>Notes</strong></td>
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<td>480 x 1440&lt;br&gt;448 x 304&lt;br&gt;332 x 316&lt;br&gt;INVALID_I4&lt;br&gt;B</td>
<td>percent</td>
<td>Percentage of ocean segments with nonzero (i.e., 4, nominal calibration maneuver) podppd_flag_seg used in the grid cell.</td>
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<td>count/meter</td>
<td>Simple average of photon noise rate</td>
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<td>count/meter</td>
<td>Simple average of surface reflected photon rate</td>
<td>photon_rate</td>
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<td>surf_prcnt_avg&lt;br&gt;cnt</td>
<td>5x[&lt;br&gt;480 x 1440&lt;br&gt;448 x 304&lt;br&gt;332 x 316]</td>
<td>percent</td>
<td>The averages of the percentages of each surface type in the grid cell ocean segment as a 5-element variable with each element corresponding to the percentage of each of the 5 surface types.</td>
<td>surf_type_prcnt</td>
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<tr>
<td>surf_prcnt_dfw&lt;br&gt;cnt</td>
<td>5x[&lt;br&gt;480 x 1440&lt;br&gt;448 x 304&lt;br&gt;332 x 316]</td>
<td>percent</td>
<td>The dfw averages of the percentages of each surface type in the grid cell ocean segment as a 5-element variable with each element corresponding to the percentage of each of the 5 surface types.</td>
<td>surf_type_prcnt</td>
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<tr>
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<td>480 x 1440&lt;br&gt;448 x 304&lt;br&gt;332 x 316&lt;br&gt;INVALID_R8&lt;br&gt;B</td>
<td>meters</td>
<td>Simple average of sea state bias</td>
<td>bin_ssbias</td>
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<td>ssb_avgentr</td>
<td>480 x 1440&lt;br&gt;448 x 304&lt;br&gt;332 x 316&lt;br&gt;INVALID_R8&lt;br&gt;B</td>
<td>meters</td>
<td>Simple all-beam average of sea state bias interpolated to center of grid cell per the ATBD</td>
<td>bin_ssbias</td>
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## 3.3 Gridding DOT for ATL23

The ATL23 processing is identical to the ATL19 section 3.2 processing except there is a 3-month span of ATL12s input.
References


ACRONYMS

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<td>ATLAS</td>
<td>ATLAS Advance Topographic Laser Altimeter System</td>
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<td>Goddard Space Flight Center</td>
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<td>ICESat-2</td>
<td>ICESat-2 Management Information System</td>
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<td>SDMS</td>
<td>Scheduling and Data Management System</td>
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<td>SIPS</td>
<td>Science Investigator-led Processing System</td>
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<td>TEP</td>
<td>Transmit Echo Pulse</td>
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GLOSSARY
### APPENDIX A: ICESat-2 Data Products

**ICESat-2 Data Products**

<table>
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<tr>
<th>File ID/Level</th>
<th>Product Name</th>
<th>Concept</th>
<th>Short Description</th>
<th>Frequency</th>
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</thead>
<tbody>
<tr>
<td>00/0</td>
<td>Telemetry Data</td>
<td>Full rate Along-track with channel info</td>
<td>Raw ATLAS telemetry in Packets with any duplicates removed</td>
<td>Files for each APID for some defined time period</td>
</tr>
<tr>
<td>01/1A</td>
<td>Reformatted Telemetry</td>
<td>Full rate Along-track with channel info</td>
<td>Parsed, partially reformatted, time ordered telemetry. Proposed storage format is NCSA HDF5.</td>
<td>Uniform time TBD minutes (1 minute?)</td>
</tr>
<tr>
<td>02/1B</td>
<td>Science Unit Converted Telemetry</td>
<td>Full rate Along-track with channel info</td>
<td>Science unit converted time ordered telemetry. Reference Range/Heights determined by ATBD Algorithm using Predict Orbit and s/c pointing. All photon events per channel per pulse. Includes Atmosphere raw profiles.</td>
<td>Uniform time TBD minutes (1 minute?)</td>
</tr>
<tr>
<td>03/2A</td>
<td>Global Geolocated Photon Data</td>
<td>Full rate Along-track with channel info</td>
<td>Reference Range/Heights determined by ATBD Algorithm using POD and PPD. All photon events per pulse per beam. Includes POD and PPD vectors. Classification of each photon by several ATBD Algorithms.</td>
<td>Uniform time TBD minutes (1 minute?)</td>
</tr>
<tr>
<td>04/2A</td>
<td>Calibrated Backscatter Profiles</td>
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<thead>
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<td>Extracted from Product 08 and 13 along track repeat heights, with slope and aspect. Snow detection flag.</td>
<td>Monthly along track product, no pointing</td>
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B.1 Average DOT
To evaluate the average DOT at the center of a grid cell we fit a plane to all the samples within the cell and the eight grid cells surrounding it and evaluate the height of the plane at the center of the center grid cell. To do this we first have to make a least-squares fit of a plane to DOT at \( N \) locations within the nine grid cells. Following Eberly (2019), given data (DOT) as a function of \( x \) and \( y \), \( h_i = f(x_i,y_i) \) at \( i=1 \) to \( N \) locations, find a least squares fit of a plane, coefficients \( a, b, \) and \( c \), to \( h_i \) with mean, \( \bar{h} = \left( \frac{1}{N} \right) \sum_{i=1}^{N} h_i \). (Also note \( \bar{x} = \left( \frac{1}{N} \right) \sum_{i=1}^{N} x_i \) and )

\[ h_i \approx ax_i + by_i + c \]  \( \text{(B1)} \)

And we want to choose \( a, b, \) and \( c \) such that the error, \( E \),

\[ E(a,b,c) = \sum_{i=1}^{N} ((ax_i + by_i + c) - h_i)^2 \]

is minimized. According to Eberle (2019) the solution is more robust, and the equations are simpler if we initially eliminate the need to determine \( c \) by taking the average of (B1) and subtracting it from (B1), to get:

and so for the deviations \( h'_i = h_i - \bar{h} \), \( x'_i = x_i - \bar{x} \), and \( y'_i = y_i - \bar{y} \)

\[ h'_i \approx ax'_i + by'_i \]  \( \text{(B2)} \)

and we will choose \( a \) and \( b \) to minimize:

\[ E(a,b) = \sum_{i=1}^{N} ((ax'_i + by'_i) - h'_i)^2 \]  \( \text{(B4)} \)

Then \( c \) will be given by \( c = \bar{h} - (a\bar{x} + b\bar{y}) \). To minimize \( E \) with respect to \( a \) and \( b \) we find \( a \) and \( b \) for which:

\[ \frac{\partial E(a,b)}{\partial a} = 2\sum_{i=1}^{N} x'_i ((ax'_i + by'_i) - h'_i) = 2\sum_{i=1}^{N} ax'_i + bx'_i y'_i - x'_h'_i = 0 \]

\[ \frac{\partial E(a,b)}{\partial b} = 2\sum_{i=1}^{N} y'_i ((ax'_i + by'_i) - h'_i) = 2\sum_{i=1}^{N} ay'_i x'_i + by'_i y'_i - y'_h'_i = 0 \]

or


Letting

\[ a = \frac{R_{yh} L_{xy} - R_{xy} L_{xy}}{L_{xx} L_{yy} - L_{xy}^2} \]
\[ b = \frac{R_{xy} L_{xx} - R_{xh} L_{xy}}{L_{xx} L_{yy} - L_{xy}^2} \]
\[ c = \bar{h} - (a \bar{x} + b \bar{y}) \]  

**Uncertainty in the Fit**

The uncertainty, \( \delta \hat{h} \), in the planar fit estimate of height \( \hat{h} \) at \( x = \hat{x} \) and \( y = \hat{y} \), is the uncertainty in

\[
\hat{h}(\hat{x}, \hat{y}) = a(\hat{x} - \bar{x}) + b(\hat{y} - \bar{y}) + \hat{h}(\bar{x}, \bar{y})
\]

\[
\hat{h} = a \hat{x} + b \hat{y} + \hat{h}_0
\]

where \( \hat{h}_0 \) is the fit at the mean data position \( \bar{x}, \bar{y} \), \( \hat{x}' = \hat{x} - \bar{x} \), and

\[
(\delta \hat{h})^2 = (\delta a \hat{x}')^2 + (\delta b \hat{y}')^2 + (\delta \hat{h}_0)^2
\]  

This is related to the RMS residual squared.

\[
(\delta \hat{h}(\bar{x}, \bar{y}))^2 = (\delta \hat{h}_0)^2 + (N-2)^{-1} E(a, b) = (N-2)^{-1} \sum_{i=1}^{N} ((a x'_i + b y'_i) - h'_i)^2
\]  

and the squared uncertainties in the slopes, \( a \), and \( b \) related to the sensitivities of the slopes to the individual \( h_i \), and the squared uncertainty in the \( h_i \). Using the first of equations (B6)
where

\[
\left(\frac{1}{L_a}\right)^2 = \left(\frac{1}{L_{xx}L_{yy} - L_{xy}^2}\right)^2
\]

and from (B5)

\[
\frac{\partial R_{xh}}{\partial h_i} = \frac{\partial}{\partial h_i} \sum_{i=1}^{N} x' h'_i = x'_i
\]

\[
\frac{\partial R_{yh}}{\partial h_i} = \frac{\partial}{\partial h_i} \sum_{i=1}^{N} y' h'_i = y'_i
\]

plugging into (B9)

\[
(\delta a)^2 = \left(\frac{1}{L_a}\right)^2 \sum_{i=1}^{N} \left((x'_i L_{yy} - y'_i L_{xy}) \delta h'_i\right)^2
\]

At this point we need to make an assumption about $\delta h'_i$. We compute an uncertainty for each ocean segment, $h_{uncrtn}$, based on the estimated degrees of freedom and height variance over the ocean segment. But this is only the uncertainty at the time and location of the measurement due to the wave induced, spatially correlated variations in height over the segment at the time of the segment. It doesn’t include the uncertainty due to temporal and spatial variability of DOT. The net uncertainty is probably better and more conservatively estimated as the residual error about the planar fit, $(\delta \hat{h}_0)^2$. Then

\[
(\delta a)^2 = (\delta \hat{h}_0)^2 \left(\frac{1}{L_a}\right)^2 \sum_{i=1}^{N} \left((x'_i L_{yy} - y'_i L_{xy})\delta h'_i\right)^2
\]

Similarly
\[
(\delta b)^2 = (\delta \hat{h}_b)^2 \left( \frac{1}{L_a} \right)^2 \sum_{i=1}^{N} \left( y'_i L_{xx} - x'_i L_{xy} \right)^2
\]  \hspace{1cm} (B13)

From (B7):

\[
(\delta \hat{h})^2 = \left( \delta \hat{b} \right)^2 (\hat{x}')^2 + (\delta b)^2 (\hat{y}')^2 + (\delta \hat{h}_b)^2
\]

\[
(\delta \hat{h})^2 = (\delta \hat{h}_b)^2 \left( \frac{1}{L_a} \right)^2 \sum_{i=1}^{N} \left( x'_i L_{yy} - y'_i L_{xy} \right)^2 (\hat{x})^2 + (\delta \hat{h}_b)^2 \left( \frac{1}{L_a} \right)^2 \sum_{i=1}^{N} \left( y'_i L_{xx} - x'_i L_{xy} \right)^2 (\hat{y}')^2 + (\delta \hat{b})^2
\]

\[
(\delta \hat{h})^2 = (\delta \hat{h}_b)^2 \left( \frac{1}{L_a} \right)^2 \left( \sum_{i=1}^{N} \left( x'_i L_{yy} - y'_i L_{xy} \right)^2 (\hat{x})^2 + \left( y'_i L_{xx} - x'_i L_{xy} \right)^2 (\hat{y}')^2 \right) + 1
\]

Expanding the sums:

\[
(\delta \hat{h})^2 = (\delta \hat{h}_b)^2 \left( \frac{1}{L_a} \right)^2 \left( \left( L_{yy} L_{xx} - L_{xy} L_{xy} \right)^2 (\hat{x})^2 + \left( L_{yy} L_{xx} - L_{xy} L_{xy} \right)^2 (\hat{y}')^2 + 1 \right)
\]  \hspace{1cm} (B14)

The square root of (B14) is the ATL19 variable \textit{dot_avgcntr_uncrtn}.

It is useful to consider the uncertainty for particular distributions of data points. If the data point locations are randomly distributed, the x position and y position are uncorrelated, Lxy is zero and (B14) becomes:

\[
\delta \hat{h}^2 = (\delta \hat{h}_b)^2 \left( \frac{1}{L_{xx} L_{yy}} \right)^2 \left( \left( L_{yy}^2 L_{xx} \right)^2 (\hat{x})^2 + \left( L_{yy} L_{xx} \right)^2 (\hat{y}')^2 + 1 \right)
\]

\[
= (\delta \hat{h}_b)^2 \left( \frac{(\hat{x})^2}{L_{xx}} + \frac{(\hat{y}')^2}{L_{yy}} + 1 \right)
\]

and the uncertainty increases away from the average data position as the ratio of \( \hat{x}' \) to the RMS spread of the data in x, \( L_{xx}^{1/2} \) and the ratio of \( \hat{y}' \) to the RMS spread of the data in y, \( L_{yy}^{1/2} \). If the x and y positions of the data are highly correlated, for example because they are on a single
straight line, the correlation coefficient, \( \left( \frac{L_{xy}^2}{L_{xx} L_{yy}} \right)^{1/2} \) approaches 1, \( L_{a} \) approaches zero, and the uncertainty becomes very large, especially for center positions well to the side of the correlated cloud of data points.

### B.2 Degree-of-Freedom-Uncertainty Weighted Average DOT

JWR – 20220621 (Not implemented in this version of Rel. 3 ATL19/23, which uses plain DOF averaging per 6/17/2022 version of the ATBD and as described in Section 3.2.4.4.2)

This basically follows the algorithm already laid out in Appendix B.1 for the unweighted case. The weights are provided by the ocean segment \( h_{\text{uncrtn}} \) values. The weights are established as the inverse square of \( h_{\text{uncrtn}} \) for each \( i^{th} \) ocean segment going into the average:

\[
W_i = \left[ \frac{1}{(h_{\text{uncrtn}})^2} \right]_i
\]

What this equation does is give stronger weight to lower values of \( h_{\text{uncrtn}} \). If an OcSeg’s \( h_{\text{uncrtn}} \) is 0.01 m, then it’s weight will be 10000; if it is 0.1 m, it has a weight of 100, etc. A standard statistical practice is to adopt a one over the sigma squared as the weight [Citation Needed].

The weights are applied from the very beginning, with the computation of weighted mean locations and heights. Analogously to the equations found just before eq. B1, we have weighted mean values:

\[
\bar{x}_w = \frac{\sum_{i=1}^{N} W_i x_i}{\sum_{i=1}^{N} W_i}
\]

\[
\bar{y}_w = \frac{\sum_{i=1}^{N} W_i y_i}{\sum_{i=1}^{N} W_i}
\]

\[
\bar{h}_w = \frac{\sum_{i=1}^{N} W_i h_i}{\sum_{i=1}^{N} W_i}
\]

(B15)

Equation B2 is very similar, except now the weighted mean values are used. A double prime has been assigned to denote the weighted de-meaned values:

\[
h''_i = h_i - \bar{h}_w, \quad x''_i = x_i - \bar{x}_w \quad \text{and} \quad y''_i = y_i - \bar{y}_w
\]

(B16)

Similar to (B5), the equations (B17) for the weighted \( L \)’s and \( R \)’s are very similar to the unweighted case, now with an additional weighting multiplier,

\[
L_{xxw} = \sum_{i=1}^{N} W_i x''_i x''_i
\]

\[
L_{yyw} = \sum_{i=1}^{N} W_i y''_i y''_i
\]

\[
L_{xyw} = \sum_{i=1}^{N} W_i x''_i y''_i
\]

(B17)
The equations that give the coefficients for the planar fit are essentially the same in form as the weighted case except we replace the $L$ & $R$ terms with the weighted versions. Using a $w$-subscript for the coefficients computed when weights are applied, i.e., $a_w$, $b_w$, $c_w$.

$$R_{xhw} = \sum_{i=1}^{N} w_i \bar{x}_i' h''_i$$

$$R_{yhw} = \sum_{i=1}^{N} w_i \bar{y}_i' h''_i$$

$$(\delta \tilde{h})_w^2 = (N/(N-2))(\sum_{i=1}^{N} w_i)^{-1} \sum_{i=1}^{N} w_i [(a_w \bar{x}_i + b_w \bar{y}_i) - h''_i]^2$$

And for

$$\left(\frac{1}{L_{aw}}\right)^2 = \left(\frac{1}{L_{xxw} L_{yyw} - L_{xyw}^2}\right)^2$$

the uncertainty, $(\delta \tilde{h}_w)$ for the weighted centered average is:

$$(\delta \tilde{h}_w)^2 = (\delta \tilde{h}_0)^2 \left\{ \left(\frac{1}{L_{aw}}\right)^2 \left[ (L_{yyw}^2 L_{xxw} - L_{yyw} L_{xyw}^2)(\bar{\bar{x}}'')^2 + (L_{xxw}^2 L_{yyw} - L_{xxw} L_{xyw}^2)(\bar{\bar{y}}'')^2 \right] + 1 \right\}$$

(B21)

Where the cell center location is denoted by: $\bar{\bar{x}}'' = x_{cell\; center} - \bar{x}_w$ and $\bar{\bar{y}}'' = y_{cell\; center} - \bar{y}_w$.  

Eberle, D., 2019, Least Squares Fitting of Data by Linear or Quadratic Structures

David Eberly, Geometric Tools, Redmond WA 98052

https://www.geometrictools.com/

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Created: July 15, 1999
Last Modified: February 14, 2019
APPENDIX C: Hierarchy of ATL12 and ATL19/23 Variables

ATL12 Inputs to ATL19/23

Segment Averages: SSH-geoid_seg-bin ssbias, lat_seg, lon_seg, bin_ssbias, geoid_seg, depth_ocn_seg, length_seg, and surf_type_prcnt.

Segment Moments: SSHvar, SSHskew, SSHkurt, swh

Segment Histogram: Y, n_photons

Segment Degrees-of-Freedom: NP_effect

3.2.4.2.1 Output ATL19/23 Averaging Over n_segs Bins

Grid Cell Averages: dot_avg, lat_avg, lon_avg, sbb_avg, geoid_avg, depth_avg, surf_prCNT_avg

Grid Cell Average Moments: dot_sigma_avg, dot_skew_avg, dot_kurt_avg, swh_avg

Grid Cell Total Histogram: dot_hist

Grid Cell Totals: n_segs, n_phrs_ttl, n_phr_srfc, length_sum

Grid cell DOT Uncertainty: dot_avg_uncrtn

3.2.4.2.2 Output ATL19/23 Averaging Weighted by Degrees-of-Freedom

Grid Cell Degree-of-Freedom Weighted Averages dot_dfw, lat_dfw, lon_dfw, sbb_dfw, geoid_dfw, depth_dfw, length_dfw, surf_prCNT_dfw

Grid Cell Degree-of-Freedom Weighted Moments dot_sigma_dfw, dot_skew_dfw, dot_kurt_dfw, swh_dfw

Grid Cell Degrees-of-Freedom and DOT Uncertainty: dof, dot_dfw_uncrttn

3.2.4.3 Output ATL19/23 Merging All-Beam, variables

All single beam gridded variables have all-beam versions, except for gridded skewness and gridded kurtosis. The all-beam variable names end in ‘_albm’.

3.2.4.4.1 Output ATL19/23 Interpolated to Bin Centers (gridcntr_lon and gridcntr_lat)

Averages at Grid Cell Center: dot_avgcntr, sbb_avgcntr

3.2.4.4.2 Output ATL19/23 Interpolated to Bin Centers (gridcntr and gridcntr_lat)

DOF Weighted Averages at Grid Cell Center: dot_dfwcntr, sbb_dfwcntr
APPENDIX D: All-beam Average Equivalencies

The all-beam parameters can be computed in the same way single beam averages are computed by merely incorporating all the ocean segments from all beams in a grid cell. However the results should be the same as the properly weighted single beam averages as described below.

The photon all-beam (*_albm) totals are computed first. For all-beam total surface reflected photons, \( n_{\text{ph}_srfc}_{albm} \) add the number of surface reflected photons, \( n_{\text{ph}_srfc} \), for each beam, and for the all-beam total of all photons in the downlink bands, \( n_{\text{phs}_ttl}_{albm} \), add the grid cell totals, \( n_{\text{phs}_ttl} \), for each beam. Further, the all-beam photon rate, \( r_{\text{ph}}_{albm} \), is equal to \( n_{\text{ph}_srfc}_{albm} \) divided by \( length_sum_{albm} \), the total of the total length of segments, \( length_sum \), for each beam. Similarly, the all-beam noise rate, \( r_{\text{noise}}_{albm} \) equals \( (n_{\text{phs}_ttl}_{albm} \text{ minus } n_{\text{ph}_srfc}_{albm}) \) divided by \( length_sum_{albm} \).

All-beam Average DOT
The average DOT of data from all six beams in the cell, \( dot_{\text{avg}}_{albm} \) should equal.

\[
dot_{\text{avg}}_{albm} = \frac{\sum [n_{\text{segs}} \times dot_{\text{avg}}]_{\text{beams 1 to 6}}}{\sum [n_{\text{segs}}]_{\text{beams 1 to 6}}}
\]

The following variables have all-beam gridded simple averages that can be calculated in the same way: depth, geoid, lat, lon, ssb, surf_prcnt and swh. The variable names all end in \(_{\text{avg}}_{albm} \).

All-beam Degree-of-Freedom Weighted Average DOT
The degree-of-freedom average DOT of all six beams in the cell, \( dot_{\text{dfw}}_{albm} \) should equal

\[
dot_{\text{dfw}}_{albm} = \frac{\sum [dof \times dot_{\text{dfwcntr}}]_{\text{beams 1 to 6}}}{\sum [dof]_{\text{beams 1 to 6}}}
\]

For each grid cell we also can compute the all-beam degrees of freedom \( dof_{albm} \), the all-beam degree-of-freedom weighted standard deviation, \( dot_{\text{sigma}}_{dfw}_{albm} \), and DOT uncertainty, \( dot_{\text{dfw}}_{\text{uncrtn}}_{albm} \).

\[
dof_{albm} = \sum [dof]_{\text{beams 1 to 6}}
\]

\[
dot_{\text{sigma}}_{dfw}_{albm} = \left(\frac{\left(\sum [dof \times (dot_{\text{sigma}}_{dfw})^2]_{\text{beams 1 to 6}}\right)}{dof_{albm}}\right)^{1/2}
\]

\[
dot_{\text{dfw}}_{albm}_{\text{uncrtn}} = dot_{\text{sigma}}_{dfw}_{albm} / (dof_{albm})^{1/2}
\]
APPENDIX E: Optimal Interpolation of ICESat-2 Dynamic Ocean Topography


Optimal Interpolation

We want to estimate or interpolate a true field of surface height or dynamic ocean topography, \( H(x) \), by an approximation \( \hat{H} \) of the form

\[
\text{(E1)}
\]

In this expression:

- \( x \) is a spatial coordinate. We could just as well have written \( H(x,y,z) \) to estimate \( H \) in 3-D.
- The number of spatial dimensions makes no difference in the following development. The coordinate \( x \) or coordinates \( x, y, z \) are just parameters.
- \( \hat{H}_j \) are observations at spatial coordinate \( x_j \).
- \( a_j(x) \) are unknown functions that we will determine. We could just as well have written \( a_j(x,y,z) \) for the 3-D case.
- So the estimate \( \hat{H}(x) \) is a linear combination of the measurements \( \hat{H}_j \). We will sometimes drop the reference to the spatial coordinate \( x \) and just write \( H, \hat{H}, \) and \( a_j \) with the understanding that these depend on the spatial coordinates.

We suppose that each measurement, \( \hat{H}_j \), consists of a true value \( H_j \) plus a measurement error \( \delta_j \):

\[
\text{(E2)}
\]

So \( H_j = H(x_j) \) is the true value of the field \( H(x) \) at the measurement point \( x_j \).

Now we form the error expression between the true field and the estimate, \( \varepsilon = H - \hat{H} \), using (E1) and (E2). We write \( \varepsilon^2 \) as

\[
\text{(E3)}
\]

At this point we introduce the idea of random variables. We consider the true value of \( H(x) \) to be an ensemble or collection of values, a random variable with some mean and variance. Similarly, the \( H_j \) are random variables. The measurement errors, \( \delta_j \), are random variables with zero mean. The coefficients \( a_j \) are not random variables. We use the notation \( E[...] \) for the expected value of a random variable. We want to determine coefficients, \( a_j \), by
minimizing $E[\varepsilon^2]$. To minimize the error with respect to the $a_j$, we set the derivative of the error with respect to each coefficient equal to zero. Assuming the errors and true heights are uncorrelated, we find the $n$ coefficients are given by a system of $n$ equations for the $n$ coefficients.

(E4)

The expression $E[\delta_j \delta_k]$ is the covariance of the measurement errors. If we make the assumption that the errors are uncorrelated then this term is zero when $j \neq k$, and we write $E[\delta_k^2] = \sigma_k^2$ for the variance of the $k^{th}$ measurement error. Then the second term in (E4) reduces to.

Now we return to the idea of the background or mean field. The expressions

\[ \frac{E[H_j H_k]}{E[H^2]} \quad \text{and} \quad \frac{E[HH_k]}{E[H^2]} \]

(E5)

would be correlations if the mean of $H$ were zero. Since we want to interpret them as correlations, we must insist that $H$ have zero mean. Also $E[H^2]$ is not a variance unless $H$ has a zero mean. So we have to modify our thinking about $H(x)$. We are free to construct any background field, $B(x)$, that we like. And we may subtract $B$ from $H$ and to get the deviations from the background:

\[ h = H - B \quad \text{and} \quad \hat{h} = \sum_{j=1}^{n} a_j \hat{h}_j \quad \text{where} \quad \hat{h}_j = H_j - B(x_j) \]

We go through the derivation of (E4) with $h$ and $\hat{h}$ instead of $H$ and $\hat{H}$ and end up with terms corresponding to (E5):

\[ \frac{E[h_j h_k]}{E[h^2]} \quad \text{and} \quad \frac{E[hh_k]}{E[h^2]} \]

(E6),

which are correlations because $h$ has zero mean. So whatever background field we construct, it must be a mean field in the sense that it leaves zero mean fluctuations when subtracted from $H(x)$ (in which case $E[h]$ equals zero and $E[\varepsilon^2]$ equals the variance of $\varepsilon$).

We now return to (E4) and consider $h$ (and $h_j$) to be fluctuations from the background field $B(x)$ such that $E[h]$ equals zero. Note that this requires subtracting $B_j$ (equal to $B(x_j) )$ from the measurements $\hat{H}_j$:

(E7)

and re-interpreting $\hat{H}$ as well, i.e., adding $B(x_j)$ to $\hat{h}$ to obtain $\hat{H}$. With the assumption of uncorrelated measurement errors equation (E4) becomes:
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(E8)

or

\[(R + D)\hat{a} = \hat{s}\]  

(E9)

where we use the matrix notation to denote:

- \(R\) = the correlation matrix of the fluctuation field between all pairs of locations where measurements are made. The \((j,k)\) entry of this \(n \times n\) symmetric matrix is \(E[h_j h_k]\) over \(E[h^2]\).
- \(D\) = the diagonal matrix with entries \(\sigma_k^2 / E[h^2]\) giving the ratio of measurement error variance to field fluctuation variance.
- \(\hat{a}\) = the vector of unknown interpolation coefficients, \(a_k, k = 1\) to \(n\).
- \(\hat{s}\) = the vector of correlations between location \(x\) and location \(x_k (k = 1\) to \(n)\) with entries \(E[h h_k]\) over \(E[h^2]\).

Equation (E9) is a system of and \(n\) equations in \(n\) unknowns. Notice that the only dependence on location \(x\) is in the right-hand side \(\hat{s}\).

**Kriging as a form of Optimal Interpolation**

Note that (E9) is very similar to D. Morison’s “Kriging_7” equation (12) for simple Kriging:

(DM12)

or

\[R_k \hat{a} = \hat{s}_k\]  

(DM12b)

where \(R_k = \begin{bmatrix} Z Z_i \end{bmatrix}\) is the covariance matrix of observations, \(\hat{a} = w_i\) is the vector of weights, and \(\hat{s}_k = \begin{bmatrix} z Z_j \end{bmatrix}\) is the vector of covariances between heights at location \(x\) and locations \(x_j\).

The Kriging equation (DM12) is similar to (E4) and the vector of weights would be the same, but the covariances are not normalized by the variance of the heights \(E[h^2]\), i.e., \(R_k = RE\begin{bmatrix} h \end{bmatrix}\) and \(\hat{s}_k = \hat{s}E\begin{bmatrix} h \end{bmatrix}\). Perhaps more importantly for our application, Kriging makes no \(a\ priori\) distinction as to measurement noise so that in Kriging, variability due to measurement noise and the natural variability of the measured variable are mixed together at the measurement locations; essentially, of equation (E9).

With ICESat-2 ATL12 SSH and ATL19/23 gridded DOT data we have calculated uncertainties, which are essentially the measurement errors for the mean SSH in ATL12 ocean segments and grid-cell averages of DOT in ATL19/23. Therefore, in principle, we can take advantage of Equation (E4) because we have a formalism for distinguishing between measurement noise, \(D\), and process variability, \(R\), in the analysis.
Elements Needed for Optimal Interpolation of ICESat-2

To summarize, what do we need to get the coefficients to use (E1) to optimally interpolate data? We need (1) a background or prior estimate of the height, $B$, as a function of the dimensional variable or variables, $x$ or $x, y, z$, (2) a square correlation matrix of the observations, $R$, (3) a diagonal matrix of measurement errors or uncertainties, $D$, and (4) a $n \times 1$ vector of correlations between the interpolant points and the observation points.

**Background, $B$** - The background field, $B$, can in principle be anything that has a mean equal to the mean of the observations. However, if the mean of the observations is used for the background, all of the variability even out to the largest scales will be included in the correlation matrix, $R$. This is unrealistic when the data domain is the global ocean, especially when we want to interpolate over a short distance, e.g., 25 km, and the physical process we want to examine has a correlation length constrained by physics. For example, the DOT measured in the Southern Ocean has nothing to do with interpolating to a 25 km grid off the coast of Greenland. In this case it makes sense to choose as a background field a climatology averaged over larger space and time scales, for which simple averaging can be done with minimal interpolation. The pertinent example is the 9-cell, 3-month averages of ICESat-2 DOT to cell centers (e.g., Figure 6, right). These can be done monthly and for almost every grid cell. And the 91-day repeat cycle of ICESat-2 is such that the 9-cell, 3-month averages have the potential for at least one satellite pass over every grid cell, so only cells under virtually perpetual cloud cover will need to have background values interpolated to them. At least initially this can be done with simple linear interpolation, and then be iterated with near optimal interpolation coefficients from prior iterations.

**Correlation Matrix, $R$** – $R$ is the covariance or correlation (the difference being the correlation matrix is covariance matrix divided by the variance) for the true data. Because we don’t actually know the true data anywhere but in particular at the point to which we want to interpolate, we have to have a model of the covariance matrix based on the covariance of observations. These can be the observations that we want to interpolate or observations in the same or similar locations made in the past. In solving equation (E4) we can use the $R$ of the actual observations to be interpolated. However, we don’t have that luxury for covariances between the data points and the points with no data, $s$. To make it generally applicable, the covariance matrix rests on a data-based model of the correlation of the variable as a function of separation in the relevant dimensions over which it is being interpolated, for example a Gaussian or decaying exponential with separation distance.

Following Kriging\textsuperscript{7 JM}, we can derive the covariance model using the observed covariogram $E[h_j h_k]$ and $E[h h_k]$ (or $[Z_i Z_i]$ and $[z Z_i]$ in the parlance of Kriging\textsuperscript{7 JM}). These are assumed to be dependent on only the separation between observation points and are therefore represented by a model of the covariogram equal to $E[h_j h_k]$ evaluated as a function of the distance, $d_{jk}$, separating observation locations $j$ and $k$:
where $C_{jk}(d_{jk})$ is the covariogram of the observations sorted by distance between observation locations.

Although $E[h_j h_k]$ and $E[h_i h_k]$ are in principle covariances of the true height values, the covariogram models are based on the observations at the observed locations. If we have a data set with the same spatial statistics as the variable we are interested in interpolating, or if there are sufficiently representative observations in the data set of interest, we can construct a sample covariogram. For every possible pair of values in the sample data, we calculate the product of the values of each sample pair as a function of the distance between each sample pair. For example, from the column vector of observation, $h_j$, we can form the symmetric covariance matrix, $C_H$:

$$C_{jk} = \begin{pmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} & \cdots & c_{nn} \end{pmatrix}$$

We also form the symmetric matrix of separation distances $D_{jk}$:

$$D_{jk} = \begin{pmatrix} d_{11} & \cdots & d_{1n} \\ \vdots & \ddots & \vdots \\ d_{n1} & \cdots & d_{nn} \end{pmatrix}$$

where $d_{jk}$ equals the distance between observation points $j$ and $k$. We then take all the values, $c_{jk}$, in the upper right half and diagonal of $C_{jk}$ paired with the corresponding $d_{jk}$, and order them in ascending values of $d_{jk}$. The resulting array of covariance values versus separation distance can be fit with a functional model of correlation versus distance. The most common forms are decaying exponentials or Gaussians.

$$C_v(d) = C_{\text{coeff}} e^{-0.5(d/L)^2}$$

or

with correlation length scale, $L$, and coefficient, $C_{\text{coeff}}$, to be adjusted to fit the covariance versus separation distance data.

Note that if we thought the correlations were different for separations in two different directions we could fit $C_{jk}(d_{xjk}, d_{yjk}) = [h_j h_k]$ as a function of $d_x$ and $d_y$ to get a model $s_x$ and $s_y$ and covariogram $V(d_x and d_y)$, exponential:
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\[ C_v\left(d_x, d_y\right) = C_{\text{off}} e^{-kd_x} e^{-kd_y} \]  \hspace{1cm} (E15)

or Gaussian:

\[ C_v\left(d_x, d_y\right) = C_{\text{off}} e^{-0.5\left(d_x/s_x\right)^2} e^{-0.5\left(d_y/s_y\right)^2} \]  \hspace{1cm} (E15)

Interpolant Correlation Vector, \( \vec{S} \) - The vector \( \vec{S} \) is the vector of correlations between location \( x \) and location \( x_k \) (\( k=1 \) to \( n \)) with entries \( E[h_i h_k] \) over \( E[h^2] \). In this case we absolutely have to have the model of covariance, because by definition there are no data for \( h \) at the interpolant point. For example, using an exponential model of the covariogram for we set

\[ C_i\left(d_j\right) = E[h h_i] = C_{\text{off}} e^{-d_j/L} \]  \hspace{1cm} (E16)

where \( C_i(d_i) \) is the model covariogram as a function of the distance, \( d_i \), between the observation locations and the location of the point to be interpolated. Note that if the constellation of measurement points was changed, for example made smaller to interpolate closer to a coastline, the same modeled correlation function from the original covariogram could be used with a smaller subset of the original observation points to form \( R \).

Measurement Error Matrix \( D \) – The matrix \( D \) is the diagonal matrix with entries \( \sigma_i^2/E[h^2] \). Because the correlation of the true \( h(x) \) with itself is 1, the diagonal elements of \( R_K = R + D \) are going to be equal to the natural variance of \( h \) as a function of location plus the measurement variance. When we use the covariogram of observations to model \( R_K \), the diagonal elements will devolve into a constant that includes natural variability and the variance due to measurement noise. Our known uncertainties from ATL12 will not enter into determining the optimal interpolation coefficients for the surface height anomalies about the background. However, we can examine the fit of the covariogram for small separation distances and extrapolate to zero separation. This will give an estimate of the average of the natural or true variance at zero separation, i.e., the diagonal elements of \( R \). The difference between the covariance extrapolation to zero separation and the observed covariance at zero separation should be similar to the uncertainties from ATL12. If it is not, we may choose to adjust the fit to the covariogram data by subtracting the average measurement noise from the covariogram data at zero separation. The resulting model \( R \) can be compared with the covariances estimated from ocean models and other observations.