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**ICE, CLOUD, and Land Elevation Satellite-2
(ICESat-2) Project**

**Algorithm Theoretical Basis Document
(ATBD)**

for

**Land-Ice Along-Track Products Part 2:
Slope-Corrected Land Ice Height Time
Series**

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**National Aeronautics and
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31

Abstract

32 This document describes the processing used to produce the ICESat-2 Slope-Corrected Land Ice
33 Height Time Series (ATL11) product from the ICESat-2 land-ice height (ATL06 project), and
34 the ATL11xo subproduct. ATL11 is produced by correcting heights in ATL06 measurements for
35 local-scale variations in surface slope using a polynomial reference surface. The remaining
36 variability in the data should mainly reflect ice-sheet surface-height changes, rather than spatial
37 variation in surface heights. ATL11 is produced for each reference ground track and pair track
38 in the ICESat-2 mission over Antarctica, Greenland, and parts of northern Eurasia and Canada.
39 The associated ATL11 subproduct provides slope-corrected heights for tracks that cross the
40 reference tracks.
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CM Foreword

43 This document is an Ice, Cloud, and Land Elevation Satellite-2 (ICESat-2) Project Science
44 Office controlled document. Changes to this document require prior approval of the Science
45 Development Team ATBD Lead or designee. Proposed changes shall be submitted in the
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49 by “may” or “can,” expectation by “will,” and descriptive material by “is.”
50 Questions or comments concerning this document should be addressed to:
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56

Preface

57

58 This document is the Algorithm Theoretical Basis Document for the TBD processing to be
59 implemented at the ICESat-2 Science Investigator-led Processing System (SIPS). The SIPS
60 supports the ATLAS (Advance Topographic Laser Altimeter System) instrument on the ICESat-
61 2 Spacecraft and encompasses the ATLAS Science Algorithm Software (ASAS) and the
62 Scheduling and Data Management System (SDMS). The science algorithm software will produce
63 Level 0 through Level 4 standard data products as well as the associated product quality
64 assessments and metadata information.

65 The ICESat-2 Science Development Team, in support of the ICESat-2 Project Science Office
66 (PSO), assumes responsibility for this document and updates it, as required, as algorithms are
67 refined or to meet the needs of the ICESat-2 SIPS. Reviews of this document are performed
68 when appropriate and as needed updates to this document are made. Changes to this document
69 will be made by complete revision.

70 Changes to this document require prior approval of the Change Authority listed on the signature
71 page. Proposed changes shall be submitted to the ICESat-2 PSO, along with supportive material
72 justifying the proposed change.

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Change History Log

Revision Level	Description of Change	SCoRe No.	Date Approved
1.0	Initial Release		
1.1	Changes for release 002. Calculate all crossovers (including near 88 S), determine the center of the y_atc search from the median of unique pair center locations.		
1.2	Changes for release 003. Add geoid and dem parameters.		
1.3	Improved the description of the polynomial coefficient writing process		
1.4	Changes for release 005: Updated geoid description, changed product name		
1.5	Changes for release 006: Added Antarctic geolocation correction, removed all references to the slope_change_rate calculation Limit y degree to 1 unless there is an adequate cross-track distribution of pair y values.		
1.6	Changes for release 007: crossing_track_data group will be published as a separate product. Filter input ATL06 data to eliminate excessively large or small heights Use <i>gelsy</i> solver for least-squares solution in reference surface calculation, check the condition number of the result to determine if the solution is valid. Fixed data types on valid_min and valid_max variable attributes Set ref_surf/dem_h values that are not between h_valid_min and h_valid_max to _invalid		

84

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1.0 INTRODUCTION

This document describes the theoretical basis and implementation of the level-3b land-ice processing algorithm for ATL11, which provides time series of surface heights. The higher-level products, providing gridded height, and gridded height change will be described in supplements to this document available in early 2020.

ATL11 is based on the ICESat-2 ATL06 Land-ice Height product, which is described elsewhere (Smith and others, 2019a, Smith and others, 2019b). ATL06 provides height estimates for 40-meter overlapping surface segments, whose centers are spaced 20 meters along each of ICESat-2's RPTs (reference pair tracks), but displaced horizontally both relative to the RPT and relative to one another because of small (a few tens of meters or less) imprecisions in the satellite's control of the measurement locations on the ground. ATL11 provides heights corrected for these offsets between the reference tracks and the location of the ATLAS measurements. It is intended as an input for high-level products, ATL14 and ATL15, which will provide gridded estimates of ice-sheet height and height change and, when used alone, as a spatially-organized product that allows easy access to height-change information derived from ICESat-2.

ATL11 employs a technique which builds upon those previously used to measured short-term elevation changes using ICESat (GLAS) repeat-track data. Where surface slopes are small and the geophysical signals are large compared to background processes (i.e., ice plains and ice shelves), some studies have subtracted the mean from a collection of height measurements from the same repeat track to leave the rapidly-changing components associated with subglacial water motion (Fricker and others, 2007) or tidal flexure (Brunt and others, 2011). In regions where off-track surface slopes are not negligible, height changes can be recovered if the mean height and an estimate of the surface slope (Smith and others, 2009) are subtracted from the data, although in these regions the degree to which the surface slope estimate and the elevation-change pattern are independent is challenging to quantify.

ICESat-2's ATL06 product provides both surface height and surface-slope information each time it overflies its reference tracks. The resulting data are similar to that from the scanning laser altimeters that have been deployed on aircraft in Greenland and Antarctica for two decades (MacGregor et al, 2021), making algorithms originally developed for these instruments appropriate for use in interpreting ATLAS data. One example is the SERAC (Surface Elevation Reconstruction and Change Detection) algorithm (Schenk & Csatho, 2012), which provides an integrated framework for the derivation of elevation change from altimetry data. In SERAC, polynomial surfaces are fit to collections of altimetry data in small (< 1 km) patches, and these surfaces are used to correct the data for sub-kilometer surface topography. The residuals to the surface then give the pattern of elevation change, and polynomial fits to the residuals as a function of time give the long-term pattern of elevation change. The ATL11 algorithm is similar to SERAC, except that (1) polynomial fit correction is formulated somewhat differently, so that the ATL11 correction gives the surface height at the fit center, not the height residual, and (2) ATL11 does not include a polynomial fit with respect to time.

2.0 BACKGROUND INFORMATION AND OVERVIEW

This section provides a conceptual description of ICESat-2's ice-sheet height measurements and gives a brief description of the derived products.

2.1 Background

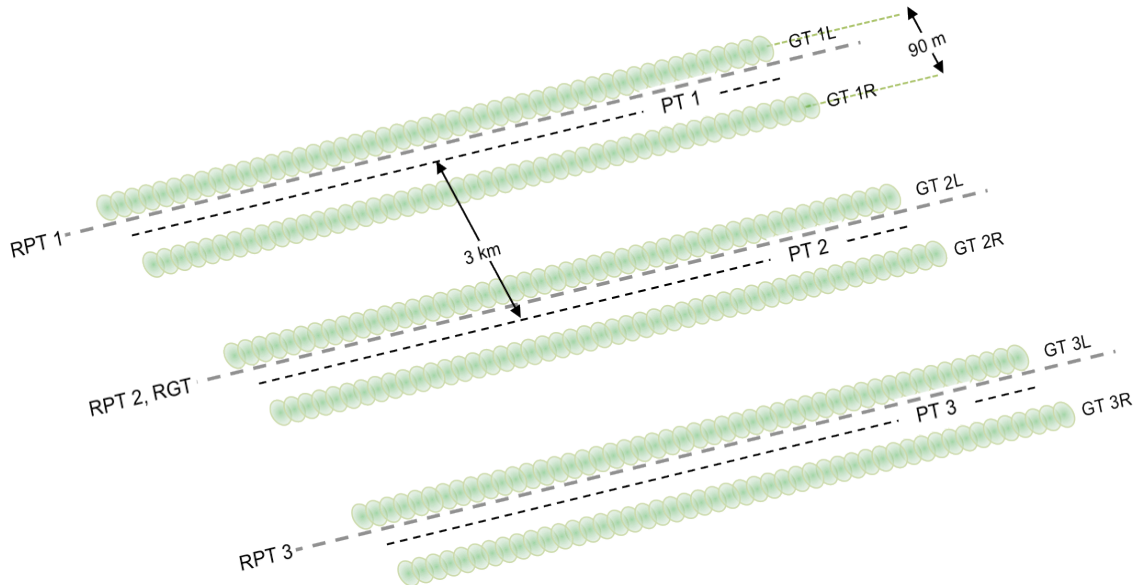
The primary goal of the ICESat-2 mission is to estimate mass-balance rates for the Earth's ice sheets. An important step in this process is the calculation of height change at specific locations on the ice sheets. In an ideal world, a satellite altimeter would exactly measure the same point on the earth on each cycle of its orbit. However, there are limitations in a spacecraft's ability to exactly repeat the same orbit and to point to the same location. These capabilities are greatly improving with technological advances but still have limits that need to be accounted for when estimating precise elevation changes from satellite altimetry data. The first ICESat mission allowed estimates of longer-term elevation rates using along-track differencing, because ICESat's relatively precise (50-150-m) pointing accuracy, precise (4-15 m) geolocation accuracy, and small (35-70-m) footprints allowed it to resolve small-scale ice-sheet topography. However, because ICESat had a single-beam instrument, its repeat-track measurements were reliable only for measuring the mean rate of elevation change, because shorter-term height differences could be influenced by the horizontal dispersion of tracks on a sloping surface.

ICESat-2 makes repeat measurements over a set of 1387 reference ground tracks (RGTs), completing a *cycle* over all of these tracks every 91 days. ICESat-2's ATLAS instrument employs a split-beam design, where each laser pulse is divided into six separate beams. The beams are organized into three *beam pairs*, with each separated from its neighbors by 3.3 km (**Figure 2-1**), each pair following a reference pair track (RPT) that is parallel to the RGT. The beams within each pair are separated by 90 m, which means that each cycle's measurement over an RPT can determine the surface slope independently, and a height difference can be derived from any two measurements of an RPT. The 90-m spacing between the laser beams in each pair is equal to twice the required RMS accuracy with which ICESat-2 can be pointed at its RPTs, which means that for most, but not all, repeat measurements of a given RPT, the pairs of beams will overlap one another. To obtain a record of elevation change from the collection of paired measurements on each RPT, some correction is still necessary to account for the effects of small-scale surface topography around the RPT in the ATL06 surface heights that appear as a result of this non-exact pointing. ATL11 uses a polynomial fit to the ATL06 measurements to correct for small-scale topography effects on surface heights that result from this non-exact pointing.

The accuracy of ICESat-2 measurements depends on the thickness of clouds between the satellite and the surface, on the reflectance, slope, and roughness of the surface, and on background noise rate which, in turn, depends on the intensity of solar illumination of the surface and the surface reflectance. It also varies from laser beam to beam, because in each of ICESat-2's beam pairs one beam (the "strong beam") has approximately four times the signal strength of the other (the "weak beam"). Parameters on the ATL06 product allow estimation of errors in each measurement, and allow filtering of most measurements with large errors due to misidentification of clouds or noise as surface returns (blunders), but to enable higher precision surface change estimates, ATL11

implements further self-consistency checks that further reduce the effects of errors and blunders.

Figure 2-1. ICESat-2 repeat-track schematic



Schematic drawing showing the pattern made by ATLAS's 6-beam configuration on the ground, for a track running from lower left to upper right. The 6 beams are grouped into 3 beam pairs with a separation between beams within a pair of 90m and a separation between beam pairs of 3.3 km. The RPTs (Reference Pair Tracks, heavily dashed lines in gray) are defined in advance of launch; the central RPT follows the RGT (Reference Ground Track, matching the nadir track of the predicted orbit). The Ground Tracks are the tracks actually measured by ATLAS (GT1L, GT1R, etc., shown by green footprints). Measured Pair Tracks (PTs, smaller dashed lines in black) are defined by the centers of the pairs of GTs, and deviate slightly from the RPTs because of inaccuracies in repeat-track pointing. The separation of GTs in each pair in this figure is greatly exaggerated relative to the separation of the PTs.

2.2 Elevation-correction Coordinate Systems

We perform ATL11 calculations using the along-track coordinate system described in the ATL06 ATBD (Smith and others, 2019b, Smith and others, 2019a). The along-track coordinate is measured parallel to the RGT, starting at each RGT's origin at the equator. The across-track coordinate is measured to the left of the RGT, so that the two horizontal basis vectors and the local vertical vector form a right-handed coordinate system.

2.3 Terminology:

Some of the terms that we will use in describing the ATL11 fitting process and the data contributing are:

RPT: Reference pair track

Cycle: ICESat-2 has 1387 distinct reference ground tracks, which its orbit covers every 91 days.

One repeat measurement of these reference ground tracks constitutes a cycle.

ATL06 segment: A 40-meter segment fit to a collection of ATL03 photon-event data, as described in the ATL06 ATBD

ATL06 pair: Two ATL06 segments from the same cycle with the same *segment_id*. By construction, both segments in the ATL06 pair have the same along-track coordinate, and are separated by the beam-to-beam spacing (approximately 90 m) in the across-track direction

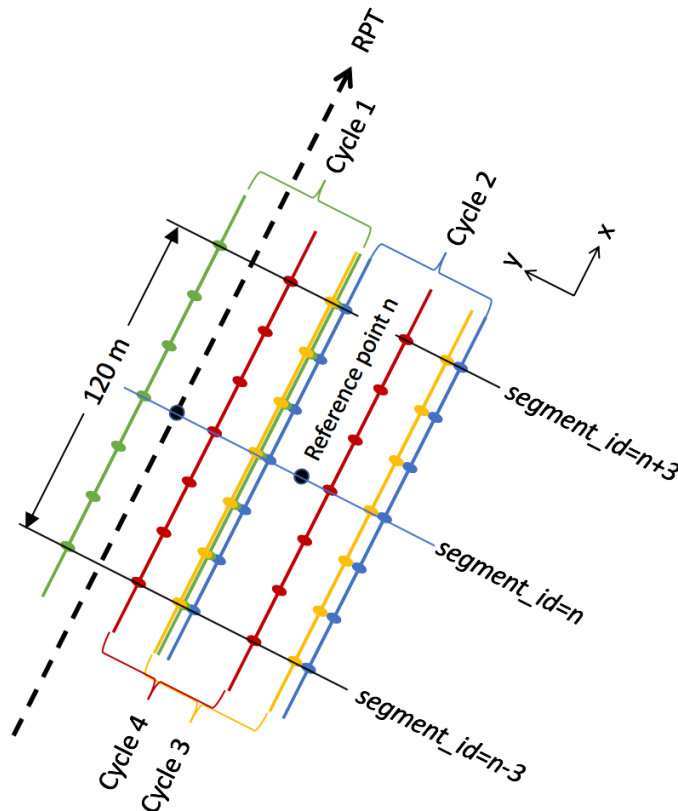
ATL11 RPT point: The expected location of each ATL11 point on the RPT, equivalent to the beginning of every third geosegment on the RPT, or the center of every third ATL06 segment.

ATL11 reference point: an *ATL11 RPT point* shifted in the across-track direction to better match the geometry of the available ATL06 data.

ATL11 fit: The data and parameters associated with a single ATL11 reference point. This includes corrected heights from all available cycles

ATL11 calculates elevations and elevation differences based on collections of segments from the same beam pair but from different cycles. ATL11 is posted every 60 m, which corresponds to every third ATL06 *segment_id*, and includes ATL06 segments spanning three segments before and after the central segment, so that the ATL11 uses data that span 120 m in the along-track direction. ATL11 data are centered on *reference points*, which has the same along-track coordinate as its central ATL06 segment, but is displaced in the across-track direction to better match the locations of the ATL06 measurements from all of the cycles present (see section 3.1.3).

Figure 2-2. ATL06 data for an ATL11 reference point



Schematic of ATL06 data for an ATL11 reference point centered on segment n , based on data from four cycles. The segment centers span 120 m in the along-track data, and the cycles are randomly displaced from the RPT in the across-track direction. The reference point has an along-track location that matches that of segment n , and an across-track position chosen to match the displacements of the cycles.

295

296 **2.4 Repeat and non-repeat cycles in the ICESat-2 mission**

297 In the early part of the ICESat-2 mission, an error in the configuration of the star trackers
298 prevented the instrument from pointing precisely at the RGTs. As a result, all data from cycles 1
299 and 2 were measured between one and two kilometers away from the RGTs, with offsets that
300 varied in time and as a function of latitude. The measurements from cycles 1 and 2 still give
301 high-precision measurements of surface height, but repeat-track measurements from ICESat-2
302 begin during cycle 3, in April of 2019. Each ATL11 granule will contain all available cycles for
303 each RGT (i.e. from cycle 3 onwards), and will contain crossovers between the repeat cycles and
304 cycles 1 and 2.

305 Outside the polar regions, ICESat-2 is pointed to minimize gaps between repeat measurements,
306 and so does not make repeat measurements over its ground tracks. ATL11 is only calculated
307 within the repeat-pointing mask (see Figure 2-3), which covers areas poleward of 60°N and
308 60°S.
309

310 **2.5 Physical Basis of Measurements / Summary of Processing**

311 Surface slopes on the Antarctic and Greenland ice sheets are generally small, with magnitudes
312 less than two degrees over 99% of Antarctica's area. Smaller-scale (0.5-3 km) undulations,
313 generated by ice flow over hilly or mountainous terrain may have amplitudes of up to a few
314 degrees. Although we expect that the surface height will change over time, slopes and locations
315 of these smaller-scale undulation are likely controlled by underlying topography and should
316 remain essentially constant over periods of time comparable with the expected ~decadal year
317 duration of the ICESat-2 mission. This allows us to use estimates of ice-sheet surface shape
318 derived from data spanning the full mission to correct for small (<130-m) differences in
319 measurement locations between repeat measurements of the same RPT, to produce records of
320 height change for specific locations. Further, we can use the surface slope estimates in ATL06 to
321 determine whether different sets of measurements for the same fit center are self-consistent: We
322 can assume that if an ATL06 segment shows a slope significantly different from others measured
323 near the same reference point it likely is in error. The combination of parameters from ATL06
324 and these self-consistency checks allows us to generate time series based on the highest-quality
325 measurements for each reference point, and our reference surface calculation lets us correct for
326 small-scale topography and to estimate error magnitudes in the corrected data.

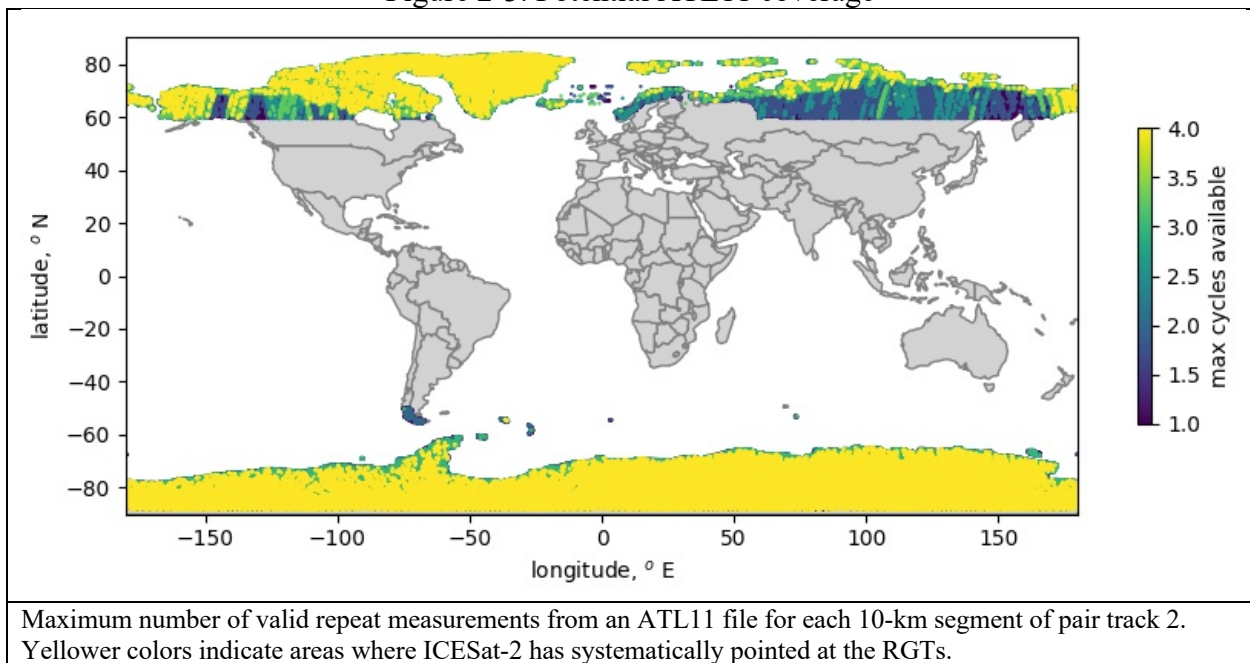
327 *2.5.1 Choices of product dimensions*

328 We have chosen a set of dimensions for the ATL11 fitting process with the goal of creating a
329 product that is conveniently sized for analysis of elevation changes, while still capturing the
330 details of elevation change in outlet glaciers. The assumption that ice-sheet surface can be

approximated by a low-degree polynomial becomes untenable as data from larger and larger areas are included in the calculation; therefore we use data from the smallest feasible area to define our reference surface, while still including enough data to reduce the sampling error in the data and to allow for the possibility that at least one or two will encounter a flat surface, which greatly improves the chances that each cycle will be able to measure surface comparable to one another. Each ATL11 point uses data from an area up to 120 m in the along-track direction by up to 130 m in the across-track direction. We have chosen the cross-track search distance ($L_{\text{search_XT}}$) to be 65 m, approximately equal to half the beam spacing, plus three times the observed 6.5 m standard deviation of the across-track pointing accuracy for cycles 3 and 4 in Antarctica. We chose the across-track search distance ($L_{\text{search_AT}}$) to be 60 m, approximately equal to $L_{\text{search_XT}}$, so that the full $L_{\text{search_AT}}$ search window spans three ATL06 segments before and after the central segment for each reference point. The resulting along-track resolution is around one third that of ATL06, but still allows 6-7 distinct elevation-change samples across a small (1-km) outlet glacier.

2.6 Product coverage

Figure 2-3. Potential ATL11 coverage



Over the vegetated parts of the Earth, ICESat-2 makes spatially dense measurements, measuring tracks parallel to the reference tracks in a strategy that will eventually measure global vegetation with a track-to-track spacing better than 1 km. Because ATL11 relies upon repeat measurements over reference tracks to allow the calculation of its reference surfaces, ATL11 is generated for ICESat-2 subregions 3-5 and 10-12 (global coverage, north and south of 60 degrees). Repeat measurements are limited to Antarctica, Greenland, and the High Arctic islands (Figure 2-3), although in other areas the fill-in strategy developed for vegetation measurements allows some repeat measurements. In regions where ICESat-2 was not pointed to the repeat track, most ATL11 reference points will provide one measurement close to the RPT. Crossover data are available for many of these points, though their distribution in time is not regular. A future

356 update to the product may provide crossover measurements for lower-latitude areas, but the
357 current product format is not designed to allow this.

3.0 ALGORITHM THEORY: DERIVATION OF LAND ICE H (T)/ATL11 (L3B)

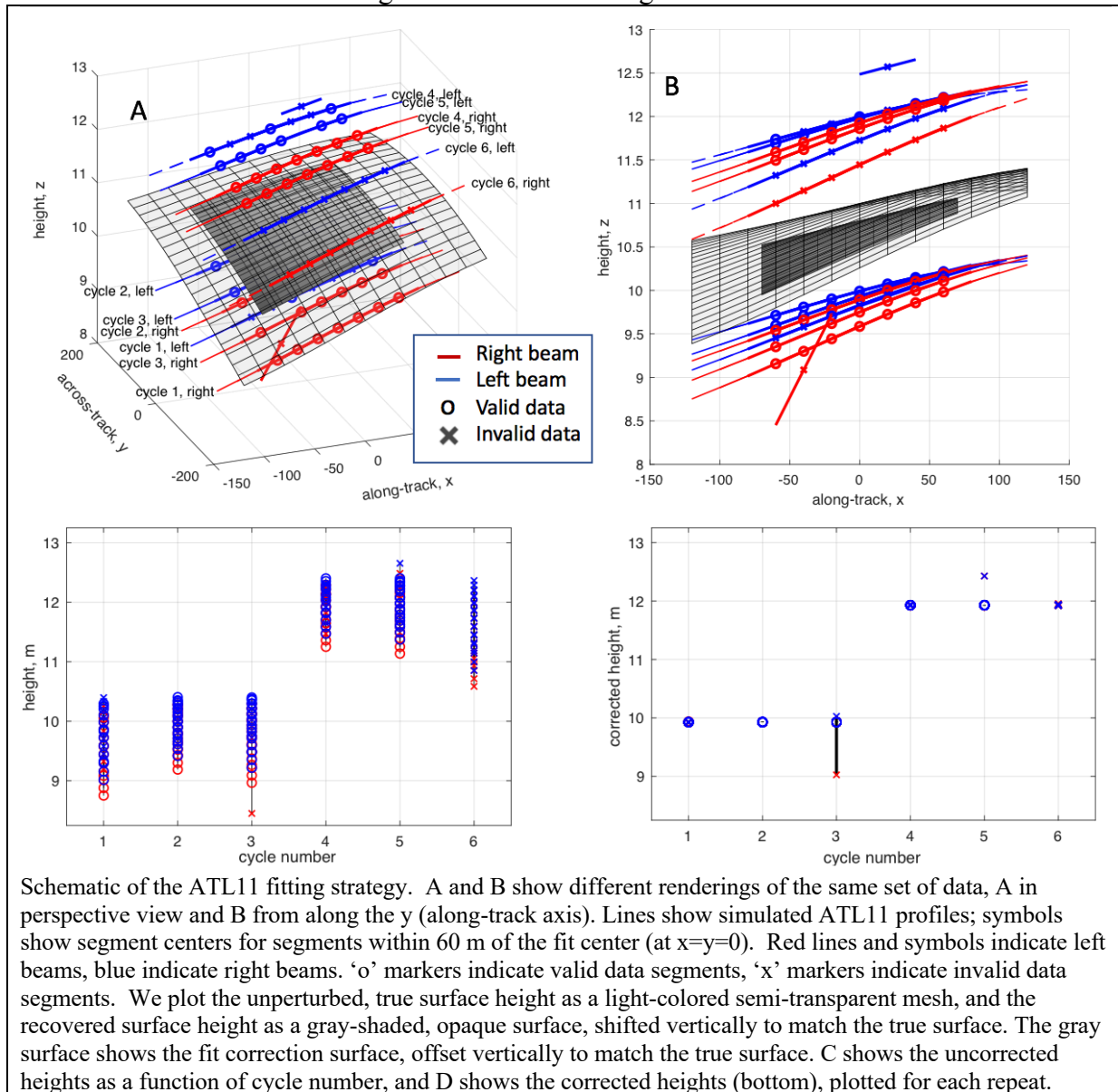
In this section, we describe in detail the algorithms used in calculating the ATL11 land-ice parameters. This product is intended to provide time series of surface heights for land-ice and ice-shelf locations where ICESat-2 operates in repeat-track mode (*i.e.* for polar ice), along with parameters useful in determining whether each height estimate is valid or a result of a variety of potential errors (see ATL06 ATBD, section 1).

ATL11 height estimates are generated by correcting ATL06 height measurements for the combined effects of short-scale (40-120-m) surface topography around the fit centers and small (up to 130-m) horizontal offsets between repeat measurements. We fit a polynomial reference surface to height measurements from different cycles as a function of horizontal coordinates around the fit centers, and use this polynomial surface to correct the height measurements to the fit center. The resulting values reflect the time history of surface heights at the reference points, with minimal contributions from small-scale local topography.

In this algorithm, for a set of reference points spaced every 60 meters along each RPT (centered on every third segment center), we consider all ATL06 segments with centers within 60 m along-track and 65 m across-track of the reference point, so that each ATL11 fit contains as many as seven distinct along-track segments from each laser beam and cycle. We select a subset of these segments with consistent ATL06 slope estimates and small error estimates, and use these segments to define a time-variable surface height and a polynomial surface-shape model. We then use the surface-shape model to calculate corrected heights for the segments from cycles not included in the initial subset. We propagate errors for each of these steps to give formal errors estimates that take into account the sampling error from ATL06, and propagate the geolocation errors with the slope of the surface-shape model to give an estimate of systematic errors in the height estimates.

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Figure 3-1. ATL11 fitting schematic



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Figure 3-1 shows a schematic diagram of the fitting process. In this example, we show simulated ATL06 height measurements for six 91-day orbital cycles over a smooth ice-sheet surface (transparent grid). Between cycles 3 and 4, the surface height has risen by 2 m. Two of the segments contain errors: The weak beam for one segment from repeat 3 is displaced downward and has an abnormal apparent slope in the x direction, and one segment from repeat 5 is displaced upwards, so that its pair has an abnormal apparent slope in the y direction. Segments falling within the across and along-track windows of the reference point (at $x=y=0$ in this plot) are selected, and fit with a polynomial reference surface (shown in gray). When plotted as a function of cycle number (panel C), the measured heights show considerable scatter but when

corrected to the reference surface (panel D), each cycle shows a consistent height, and the segments with errors are clearly distinct from the accurate measurements.

3.1 Input data editing

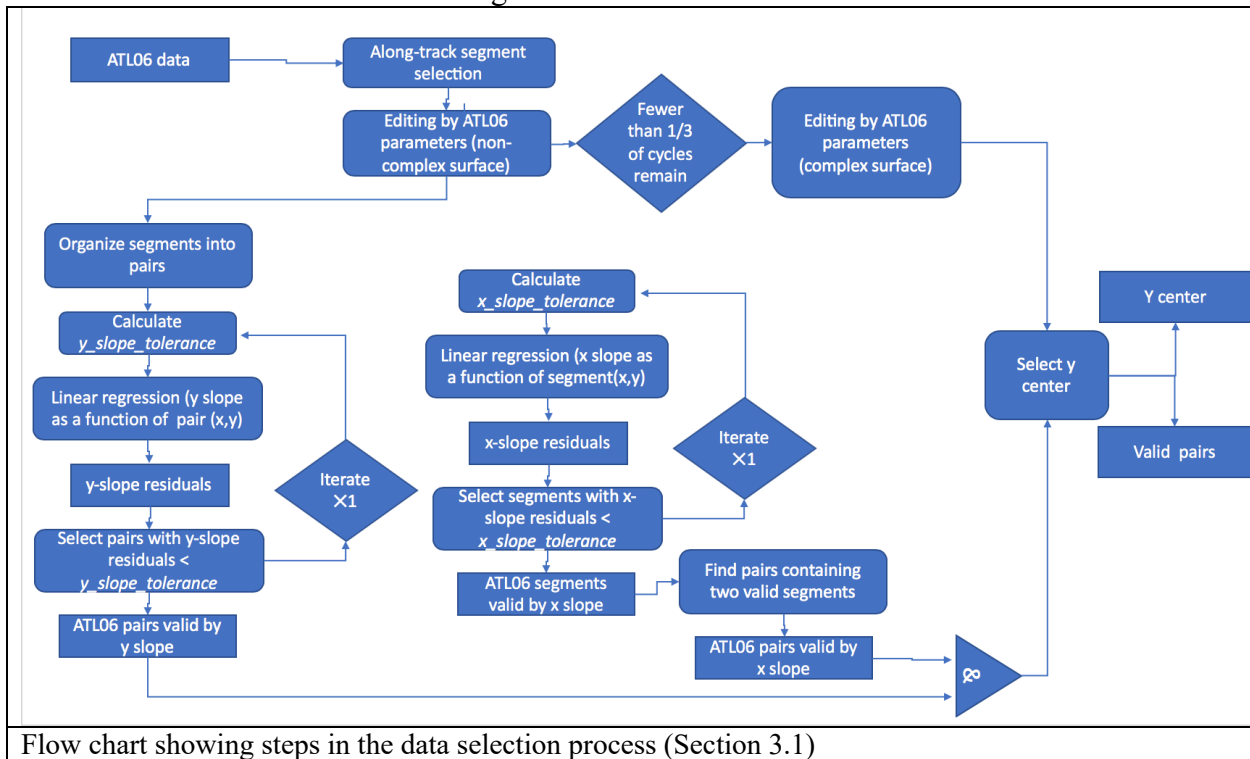
Each ATL06 measurement includes location estimates, along- and across-track slope estimates, and PE (Photon-Event)-height misfit estimates. To calculate the reference surface using the most reliable subset of available data, we perform tests on the surface-slope estimates and error statistics from each ATL06-pair to select a self-consistent set of data. These tests determine whether each pair of measurements is *valid* and can be used in the reference-shape calculation or is *invalid*. Segments from invalid pairs may be used in elevation-change calculations, but not in the reference-shape calculation.

A complete flow chart of the data-selection process is shown in Figure 3-2, and the parameters used to make these selections and their values are listed in Table 3-1.

Table 3-1 Parameter Filters to determine the validity of segments for ATL11 estimates

complex_surface_flag	Segment parameter	Filter strategy	Section
0	<i>ATL06_quality_summary</i>	<i>ATL06_quality_summary</i> =0 (indicates high-quality segments)	3.1.1
1	<i>SNR_significance</i>	<i>SNR_significance</i> < 0.02 (indicates low probability of surface-detection blunders)	3.1.1
0 or 1	Along-track differences	Minimum height difference between the endpoints of a segment and the middles of its neighbors must be < 2 m (for smooth surfaces) or < 10 m (for complex surfaces)	3.1.1
0 or 1	<i>h_li_sigma</i>	<i>h_li_sigma</i> < max(0.05, 3*median(<i>h_li_sigma</i>))	3.1.1
0 or 1	Along-track slope	$ r_slope_x < 3 \text{ slope_tolerance_x}$	3.1.2
0 or 1	Across-track slope	$ r_slope_y < 3 \text{ slope_tolerance_y}$	3.1.2
0 or 1	Segment location	$ x_atc-x_0 < L_search_XT$ $ y_atc-y_0 < L_search_AT$	3.1.3

Figure 3-2. Data selection



3.1.1 Input data editing using ATL06 parameters

For each reference point, we collect all ATL06 data from all available repeat cycles that have *segment_id* values within ± 3 of the reference point (inclusive) and that are on the same *rgt* and pair track as the reference point. The *segment_id* criterion ensures that the segment centers are within ± 60 m of the reference point in the along-track direction. We next check that the ATL06 data are close to the pre-defined reference track, by rejecting all ATL06 segments that are more than 500 m away from the nominal pair across-track coordinates (-3200, 0, and 3200 meters for right, center, and left pairs, respectively). This removes data that were intentionally or accidentally collected with ATLAS pointed off nadir (i.e. for calibration scan maneuvers).

ATL06 contains some segments with signal-finding blunders (Smith et al., 2019). To avoid having these erroneous segments contaminate ATL11, we filter using one of two sets of tests, depending on surface roughness. We identify high-quality ATL06 segments, using parameters that depend on whether the surface is identified as smooth or rough, as follows:

- 1) For smooth ice-sheet surfaces, we use the ATL06 *ATL06_quality_summary* parameter, combined with a measure of along-track elevation consistency, *at_min_dh*, that is calculated as part of ATL11. *ATL06_quality_summary* is based on the spread of the residuals for each segment, the along-track surface slope, the estimated error, and the signal strength. Zero values indicate that no error has been found. We define the along-track consistency parameter *at_min_dh* as the minimum absolute difference between the heights of the endpoints of each segment and the center heights of the previous and subsequent segments. Its value will be small if a segment's height and slope are consistent with at least one of its neighbors. For smooth

surfaces, we require that the *at_min_dh* values be less than 2 m. Over smooth ice-sheet surfaces, the 2-m threshold eliminates most blunders without eliminating a substantial number of high-quality data points.

2) For rough, crevassed surfaces, the smooth-ice strategy may not identify a sufficient number of pairs for ATL11 processing to continue. If fewer than one third of the original cycles remain after the smooth-surface criteria are applied, we relax our criteria, using the signal-to-noise ratio (based on the ATL06 *segment_stats/snr_significance* parameter) to select the pairs to include in the fit, and require that the *at_min_dh* values be less than 10 m. If we relax the criteria in this way, we mark the reference point as having a complex surface using the *ref_surf/complex_surface_flag*, which limits the degree of the polynomial used in the reference surface fitting to 0 or 1 in each direction.

For either smooth or rough surfaces, we perform an additional check using the magnitude of *h_li_sigma* for each segment. If any segment's value is larger than three times the maximum of 0.05 m and the median *h_li_sigma* for the valid segments for the current reference point, it is marked as invalid. The limiting 0.05 m value prevents this test from removing high-quality data over smooth ice-sheet surfaces, where errors are usually small.

3) To help eliminate a small minority of blunders in ATL06 that are not caught by the other tests, any ATL06 with a *h_li* less than -460 m or greater than 8400 m is marked as invalid. These limits are chosen based on the range of height values observed for land surfaces in ATL08 as of release 007.

Each of these tests applies to values associated with ATL06 segments. When the tests are complete, we check each ATL06 pair (*i.e.* two segments for the same along-track location from the same cycle) and if either of its two segments has been marked as invalid, the entire pair is marked as invalid.

3.1.2 Input data editing by slope

The segments selected in 3.1.1 may include some high-quality segments and some lower-quality segments that were not successfully eliminated by the data-editing criteria. We expect that the ATL06 slope fields (*dh_fit_dx*, and *dh_fit_dy*) for the higher-quality data should reflect the shape of an ice-sheet surface with a spatially consistent surface slope around each reference point, but that at least some of lower-quality data should have slope fields that outliers relative to this consistent surface slope. In this step, we assume that the slope may vary linearly in *x* and *y*, and so use residuals between the slope values and a regression of the slope values against *x* and *y* to identify the data with inconsistent slope values. The data with large residuals are marked as *invalid*.

Starting with valid pairs from 3.2.1, we first perform a linear regression between the *y* slopes of the pairs and the pair-center *x* and *y* positions. The residuals to this regression define one *y_slope_residual* for each pair. We compare these residuals against a *y_slope_tolerance*:

$$y_slope_tolerance = \max(0.01, 3 \text{ median } (dh_fit_dy_sigma), 3 \text{ RDE } (y_slope_residuals)) \quad 1$$

Here RDE is the Robust Difference Estimator, equal to half the difference between the 16th and 84th percentiles of a distribution, and the minimum value of 0.01 ensures that this test does not remove high-quality segments in regions where the residuals are very consistent. If any pairs

have a $y_slope_residual$ greater than $y_slope_tolerance$, we remove them from the group of valid pairs, then repeat the regression, recalculate $y_slope_tolerance$, and retest the remaining pairs. We then return to the pairs marked as *valid* from 3.1.1, and perform a linear regression between the x slopes of the segments within the pairs and the segment-center x and y positions. The residuals to this regression define one $x_slope_residual$ for each segment. We compare these residuals against an $x_slope_tolerance$, calculated in the same way as (1), except using segment x slopes and residuals instead of pair y slopes. As with the y regression, we repeat this procedure once if any segments are eliminated in the first round. After both the x and y regression procedures are complete, each pair of segments is marked as *valid* if both of its x residuals are smaller than $slope_tolerance_x$ and its y residual is smaller than $slope_tolerance_y$.

3.1.3 Spatial data editing

The data included in the reference-surface fit fall in a “window” defined by a $2L_{search\ XT}$ by $2L_{search\ AT}$ rectangle, centered on each reference point. Because the across-track location of the repeat measurements for each reference point are determined by the errors in the repeat track pointing of ATLAS, a data selection window centered on the RPT in the y direction will not necessarily capture all of the available cycles of data. To improve the overlap between the window and the data, we shift the reference point in the y direction so that the window includes as many valid beam pairs as possible. We make this selection after the parameter-based (3.1.1) and slope-based (3.1.2) editing steps because we want to maximize the number high-quality pairs included, without letting the locations of low-quality segments influence our choice of the reference-point shift.

We select the across-track offset for each reference point by searching a range of offset values, δ , around the RPT to maximize the following metric:

$$M(\delta) = \frac{[number\ of\ unique\ valid\ pairs\ entirely\ contained\ in\ \delta \pm L_{search\ XT}] + [number\ of\ unpaired\ segments\ contained\ in\ \delta \pm L_{search\ XT}]}{100} \quad 2$$

Maximizing this metric allows the maximum number of pairs with two valid segments to be included in the fit, while also maximizing the number of segments included close to the center of the fit. If multiple values of δ have the same M value we choose the median of those δ values.

The across-track coordinate of the adjusted reference point is then $y_0 + \delta_{max}$, where y_0 is the across-track coordinate of the unperturbed reference point. After this adjustment, the segments in pairs that are contained entirely in the across-track interval $\delta \pm L_{search\ XT}$ are identified as *valid* based on the spatial search.

The location of the adjusted reference point is reported in the data group for each pair track, with corresponding local coordinates in the *ref_surf* subgroup: */ptx/ref_surf/x_atc*, */ptx/ref_surf/y_atc*.

3.2 Reference-Surface Shape Correction

To calculate the reference-surface shape correction, we construct the background surface shape from valid segments selected during 3.1 and 3.2, using a least-squares inversion that separates surface-shape information from elevation-change information. This produces surface shape-corrected height estimates for cycles containing at least one valid pair, and a surface-shape

509 model that we use in later steps (3.4, 3.6) to calculate corrected heights for cycles that contain no
 510 valid pairs and to calculate corrected heights for crossing tracks.

511 3.2.1 Reference-surface shape inversion

512 The reference-shape inversion solves for a reference surface and a set of corrected-height values
 513 that represent the time-varying surface height at the reference point. The inversion involves
 514 three matrices:

515 (i): a polynomial surface shape matrix, **S**, that describes the functional basis for the spatial part of
 516 the inversion:

$$\mathbf{S} = \left[\left(\frac{x - x_0}{l_0} \right)^p \left(\frac{y - y_0}{l_0} \right)^q \right] \quad 3$$

517 Here x_0 and y_0 are equal to the along-track coordinates of the adjusted reference point,
 518 `/ptx/ref_surf/x_atc` and `/ptx/ref_surf/y_atc`, respectively. **S** has one column for each permutation
 519 of p and q between zero and the degree of the surface polynomial in each dimension, but does
 520 not include a $p=q=0$ term. The degree in the along-track direction is chosen to be no more than
 521 3 and no more than 1 less than the number of distinct x values in any cycle. The degree in the
 522 across-track direction is chosen to be no more than 2 and to be no more than the number of
 523 distinct pair-center y values, with the further (usually redundant) restriction that if the standard
 524 deviation of selected pair y values is less than twice the across-track geolocation accuracy, the
 525 across-track degree is no more than 1. For both components, distinct values are defined at a
 526 resolution of 20 m. The scaling factor, l_0 , ensures that the components of **S** are on the order of 1,
 527 which improves the numerical accuracy of the computation. We set $l_0=100$ m, to approximately
 528 match the intra-pair beam spacing.

529 (ii): a matrix that encodes the repeat structure of the data, that accounts for the height-change
 530 component of the inversion:

$$\mathbf{D} = [\delta(i, 1), \delta(i, 2), \dots, \delta(i, N)] \quad 4$$

531 Here δ is the delta function, equal to 1 when its arguments are equal, zero otherwise, and i is an
 532 index that increments by one for each distinct cycle in the selected data.

533 The surface shape and height time series are estimated by forming a composite design matrix, **G**,
 534 where

$$\mathbf{G} = [\mathbf{S}, \mathbf{D}], \quad 5$$

535 and a covariance matrix, **C**, containing the squares of the segment-height error estimates on its
 536 diagonal. The surface-shape polynomial and the height changes are found:

$$[\mathbf{s}, \mathbf{z}_c] = \mathbf{G}^{-g} \mathbf{z} \quad 6$$

$$\text{where} \quad \mathbf{G}^{-g} = [\mathbf{G}^T \mathbf{C}^{-1} \mathbf{G}]^{-1} \mathbf{G}^T \mathbf{C}^{-1}$$

537 The notation $[\]^{-1}$ designates the inverse of the quantity in brackets, and $\mathbf{z}$ is the vector of segment
 538 heights. The parameters derived in this fit are **s**, a vector of surface-shape polynomial
 539 coefficients, and $\mathbf{z}_c$, a vector of corrected height values, giving the height at (lat_0, lon_0) as
 540 inferred from the height measurements and the surface polynomial. The matrix $\mathbf{G}^{-g}$ is the
 541 generalized inverse of **G**. The values of **s** are reported in the `ref_surf/poly_ref_surf` parameter, as
 542 they are calculated from (6), with no correction made for the scaling in (3).

3.2.2 Misfit analysis and iterative editing

If blunders remain in the data input to the reference-surface calculation, they can lead to inaccurate reference surfaces. To help remove these blunders, we iterate the inversion procedure in 3.2.1, eliminating outlying data points based on their residuals to the reference surface.

To determine whether outliers may be present, we calculate the chi-squared misfit between the data and the fit surface based on the data covariance matrix and the residual vector, r :

$$\chi^2 = r^T \mathbf{C}^{-1} r \quad 7$$

To determine whether this misfit statistic indicates consistency between the polynomial surface and the data we use a P statistic, which gives the probability that the given χ value would be obtained from a random Gaussian distribution of data points with a covariance matrix $\mathbf{C}$. If the probability is less than 0.025, we perform some further filtering/editing: we calculate the RDE of the scaled residuals, eliminate any pairs containing a segment whose scaled residual magnitude is larger than three times that value, and repeat the remaining segments.

After each iteration, any column of $\mathbf{G}$ that has a uniform value (i.e. all the values are the same) is eliminated from the calculation, and the corresponding value of the left-hand side of equation 7 is set to zero. Likewise, if the inverse problem has become less than overdetermined (i.e., the number of data is smaller than the number of unknown values they are constraining), the polynomial columns of $\mathbf{G}$ are eliminated one by one until the number of data is greater than the number of unknowns. Columns are eliminated in descending order of the sum of x and y degrees, and when there is a tie between columns based on this criterion, the column with the larger y degree is eliminated first.

In some cases, the data editing process results in a $\mathbf{G}$ matrix that is of less than full rank (i.e. than some of the columns are linearly dependent), as determined by the least-squares solution algorithm *gelsy*. In these cases, the solution for the reference point is marked as *invalid* and the iterations end.

This fitting procedure is continued until no further segments are eliminated. If more than three complete cycles that passed the initial editing steps are eliminated in this way, the surface is assumed to be too complex for a simple polynomial approximation, and we proceed as follows:

(i) the fit and its statistics are reported based on the complete set of pairs that passed the initial editing steps (valid pairs), using a planar ($x_degree = y_degree = 1$) fit in x and y .

(ii) the *ref_surf/complex_surface_flag* is set to 1.

The misfit parameters are reported in the *ref_surf* group: The final chi-squared statistic is reported as *ref_surf/misfit_chi2r*, equal to the chi-squared statistic divided by the number of degrees of freedom in the solution; the final RMS of the scaled residuals is reported as *ref_surf/misfit_rms*.

3.3 Reference-shape Correction Error Estimates

We first calculate the errors in the corrected surface heights for segments included in the reference-surface fit. We form a second covariance matrix, $\mathbf{C}_1$, whose diagonal elements are the maximum of the squares of the segment errors and $\langle r^2 \rangle$. We estimate the covariance matrix for the height estimates:

$$\mathbf{C}_m = \mathbf{G}^{-g} \mathbf{C}_1 \mathbf{G}^{-gT} \quad 8$$

The square roots of the diagonal values of $\mathbf{C}_m$ give the estimated errors in the surface-polynomial and height estimates due to short-spatial-scale errors in the segment heights. If there are N_{coeff} coefficients in the surface-shape polynomial, and $N_{shape-cycles}$ cycles included in the surface-shape fit, then the first N_{coeff} diagonal elements of $\mathbf{C}_m$ give the square of the errors in the surface-shape polynomial and the last $N_{shape-cycles}$ give the errors in the surface heights for the cycles included in the fit. The portion of $\mathbf{C}_m$ that refers only to the surface shape and surface-shape change components is $\mathbf{C}_{m,s}$.

3.4 Calculating corrected height values for repeats with no selected pairs

Once the surface polynomial has been established from the edited data set, corrected heights are calculated for the unselected cycles (*i.e.* those from which all pairs were removed in the editing steps): For the segments among these cycles, we form a new design matrix, $\mathbf{S}$ and multiply it by $\mathbf{s}$ to give the surface-shape correction:

$$\mathbf{z}_c = \mathbf{z} - \mathbf{S}\mathbf{s} \quad 9$$

Here $\mathbf{s}$ is the surface-shape polynomial. This gives up to fourteen corrected-height values per unselected cycle. From among these, we select the segment with the minimum error, as calculated in the next step.

The height errors for segments from cycles not included in the surface-shape fit are calculated:

$$\sigma_{z,c}^2 = \text{diag}(\mathbf{S}\mathbf{C}_{m,s}\mathbf{S}^T) + \sigma_z^2 \quad 10$$

Here σ_z is the error in the segment height, and $\sigma_{z,c}$ is the error in the corrected height. The results of these calculations give a height and a height error for each unselected segment. To obtain a corrected elevation for each repeat that contains no selected pairs, we identify the segment from that repeat that has the smallest error estimate, and report the value z_c as that repeat's *ptx/h_corr*, and use $\sigma_{z,c}$ as its error (*/ptx/h_corr_sigma*).

3.5 Calculating systematic error estimates

The errors that have been calculated up to this point are due to errors in fitting segments to photon-counting data and due to inaccuracies in the polynomial fitting model. Additional error components can result from more systematic errors, such as errors in the position of ICESat-2 as derived from POD, and pointing errors from PPD. These are estimated in the ATL06 *sigma_geo_xt*, *sigma_geo_at*, and *sigma_geo_r* parameters, and their average for each repeat is reported in the *cycle_stats* group under the same parameter names. The geolocation component of the total height is the product of the geolocation error and the surface slope, added in quadrature with the vertical height error:

$$\sigma_{h,systematic} = \left[\left(\frac{dh}{dx} \sigma_{geo,AT} \right)^2 + \left(\frac{dh}{dy} \sigma_{geo,XT} \right)^2 + \sigma_{geo,r}^2 \right]^{1/2} \quad 11$$

For selected segments, which generally come from pairs containing two high-quality height estimates, dh/dy is estimated from the ATL06 *dh_fit_dy* parameter. For unselected segments, it is based on the y component of the reference-surface slope, as calculated in section 4.2.

The error for a single segment's corrected height is:

$$\sigma_{h,total} = [\sigma_{h,systematic}^2 + \sigma_{h,c}^2]^{1/2} \quad 12$$

This represents the total error in the surface height for a single corrected height. In most cases, error estimates for averages of ice-sheet quantities will depend on errors from many segments

from different reference points, and the spatial scale of the different error components will need to be taken into account in error propagation models. To allow users to separate these effects, we report both the uncorrelated error, $/ptx/h_corr_sigma$, and the component due only to systematic errors, $/ptx/h_corr_sigma_systematic$. The total error is the quadratic sum of the two, as described in equation 13.

3.6 Calculating shape-corrected heights for crossing-track data

Locations where groundtracks cross provide opportunities to check the accuracy of measurements by comparing surface-height estimates between the groundtracks, offers the opportunity to generate elevation-change time series that have more temporal detail than the 91-day repeat cycle can offer for repeat-track measurements, and provides data from tracks and cycles during which ATLAS was not pointed at the RGTs (e.g. cycles 1 and 2).

At these crossover points, we use the reference surface calculated in 3.5 to calculate corrected elevations for the crossing tracks. We refer to the track for which we have calculated the reference surface as the *datum* track, and the other track as the *crossing* track. To calculate corrected surface heights for the crossing ICESat-2 orbits, we first select all data from the crossing orbit within a distance L_search_XT of the updated reference point on the datum track. For most datum reference points, this will yield no crossing data, in which case the calculation for that datum point terminates. If crossing data are found, we then calculate the coordinates of these points in the reference point's along-track and across-track coordinates. This calculation begins by transforming the crossing-track data into local northing and easting coordinates relative to the datum reference-point location:

$$\delta N_c = \frac{\pi R_e}{180} (lat_c - lat_d) \quad 13$$

$$\delta E_c = \frac{\pi R_e}{180} (lon_c - lon_d) \cos(lat_c)$$

Here (lat_d, lon_d) are the coordinates of the adjusted datum reference point, (lat_c, lon_c) are the coordinates of the points on the crossing track, and R_e is the local radius of the WGS84 ellipsoid. We then convert the northing and easting coordinates into along-track and across-track coordinates based on the azimuth ϕ of the datum track:

$$\begin{aligned} x_c &= \delta N_c \cos(\phi) + \delta E_c \sin(\phi) \\ y_c &= \delta N_c \sin(\phi) - \delta E_c \cos(\phi) \end{aligned} \quad 14$$

Using these coordinates, we proceed as we did in 3.4 and 3.5: we generate S_k and S_{kt} matrices, use them to correct the data and to identify the data point with the smallest error for each crossing cycle. We report the time, error estimate, and corrected height for the minimum-error datapoint from each cycle, as well as the location, pair, and track number for the datum track. Because the crossing angles between the tracks are oblique at high latitudes, a particular crossing track may appear in a few subsequent datum points; in these cases, we expect that the error estimates should vary with the distance between the crossing track and the datum track, so that the point with the minimum error should correspond to the precise crossing location of the two tracks.

To help evaluate the quality of crossing-track data we calculate the *along_track_rss* parameter for each crossing-track measurement. This parameter gives the RSS of the differences between each segment's endpoint heights and the heights of the previous and subsequent segments. A segment that is consistent with the previous and next segments in slope and

elevation will have a small value for this parameter, a segment that is inconsistent (and thus potentially in error) will have a large value. Crossing-track measurements that have values greater than 10 m are excluded from ATL11 and do not appear in the dataset.

Data for crossing tracks are provided in the *ATL11xo* subproduct. We anticipate that crossover data will be of most use to researchers studying regional averages of height change, and that collecting crossover estimates from multiple tracks by reading the data from each individual granule would be inefficient under these circumstances. To improve the spatial organization of this product, we provide it in 200x200-km tiles defined in projected coordinates, with one file for each tile and for each ICESat-2 repeat cycle. Users will be able to locate the tiles associated with their area of interest based on the tile location encoded in the filename.

Unlike the main ATL11 product, the ATL11xo product is not fully updated as new data become available. When a new release of ATL11 is generated, the ATL11xo is generated for the full range of cycles available at the start of that release, using the reference surface determined at the start of the release. Then, as further cycles of data become available, the ATL11xo subproduct is generated for the cycles in each new batch of data and for any cycles that were not completed in the previous batch, with an updated version number corresponding to the version number of the along-track ATL11 product. As an example, we expect release 007 version 01 of ATL11 to contain along-track data from full cycles 03-26, and part of cycle 27, so Release 007 version 01 of ATL11xo will provide tiles of crossing-track data for cycles 01-27. Assuming that the next batch of data becomes available in September 2025, a version 02 of ATL11 will be generated spanning along-track cycles 1-27, with partial coverage of cycle 28, and ATL11xo version 02 will be released for cycles 27 and 28.

3.7 Calculating parameter averages

ATL11 contains a variety of parameters that mirror parameters in ATL06, but are averaged to the 140-m ATL11 resolution. Except where noted otherwise, these quantities are weighted averages of the corresponding ATL06 values. For selected pairs (i.e. those included in the reference-surface fit), the parameters are averaged over the selected segments from each cycle, using weights derived from their formal errors, h_li_sigma . The parameter weighted average for the N_k segments from cycle k is then:

$$\langle q \rangle = \frac{\sum_{i=1}^{N_k} |\sigma_i^{-2}| q_i}{\sum_{i=1}^{N_k} |\sigma_i^{-2}|} \quad 15$$

Here q_i are the parameter values for the segments. For repeats with no selected pairs, recall that the corrected height for only one segment is reported in */ptx/h_corr*; for these, we simply report the corresponding parameter values for that selected segment.

3.8 Output data editing

The output data product includes cycle height estimates only for those cycles that have non-systematic error estimates (*/ptx/h_corr_sigma*) less than 15 m. All other heights (and their errors) are reported as *invalid*. In addition, *h_corr* values that are outside the range of [-460,

+8400] (the approximate values of terrestrial height values observed in ATL08, release 007) are marked as *invalid*, as are *ref_surf/dem_h* values that fall outside this range.

3.9 Antarctic geolocation biases

As part of understanding apparent track-to-track variations in surface-height-change rates that we had seen in the gridded products, we performed a crossover analysis of ATL06 data in Antarctica and Greenland. This analysis showed that in the Antarctic data there are systematic spot-to-spot elevation biases that vary on periods of a few months, but no corresponding bias variations in Greenland. Analysis of the spot-to-spot pattern in these biases shows that they are consistent with biases in the geolocation for the whole array, varying slowly in the along- and across-track direction, which leads to height biases that are largest for the outlying spots in the array, particularly at times when the array was pointed away from nadir. In release 6 of ATL11, these biases are corrected, so that the Antarctic product shows a much smaller systematic height variation between the left and right beam pairs. Expected maximum changes in reported surface height are on the order of 3-5 cm in magnitude, opposite in sign between left and right beam pairs. We have introduced the *dh_geoloc* parameter to report the magnitude of the correction for each reference point. This parameter is zero for all data except those collected in the Antarctic.

3.9.1 Estimating Antarctic geolocation biases

The relative displacement between the measured spot location and ICESat-2 is specified by the segment azimuth, ϕ_{sg} , the azimuth of the beam, ϕ_{ref} , the surface height, h_{LI} , the elevation angle of the satellite from each spot, θ_{sat} , and the ~511-km height of ICESat-2 above the reference ellipsoid (H_{sat}). The vector from the satellite to each spot, in along-track coordinates, is

$$\begin{bmatrix} x_{spot} & y_{spot} & dh_{surf-sat} \end{bmatrix} = \begin{bmatrix} -\rho \cos(\phi_{seg} - \phi_{ref}) & -\rho \sin(\phi_{seg} - \phi_{ref}) & h_{li} - H_{sat} \end{bmatrix} \quad 16$$

where

$$\rho = (H_{sat} - h_{LI}) \tan(\theta_{sat}).$$

The right triangle between the nadir point below the surface ($x=y=0$), the satellite, and the spot locations defines the relation between the spot location and the estimated surface height:

$$(H_{sat} - h_{surf})^2 + x_{spot}^2 + y_{spot}^2 = R^2 \quad 17$$

The differential of this equation with respect to h_{surf} , x_{spot} and y_{spot} gives:

$$dh_{surf} = \frac{x_{spot}}{(H_{sat} - h_{surf})} \delta x_{spot} + \frac{y_{spot}}{(H_{sat} - h_{surf})} \delta y_{spot} \quad 18$$

This expresses the relation between errors in the x and y components of spot position and errors in the estimated surface height. At a crossover point, the estimated height difference between the two measurements will include an error component due to spot mislocation:

$$h_2 - h_1 = \left(\frac{x_{spot,2} - x_{spot,1}}{H_{sat} - h_{surf}} \right) \delta x_{spot} + \left(\frac{y_{spot,2} - y_{spot,1}}{H_{sat} - h_{surf}} \right) \delta y_{spot} \quad 19$$

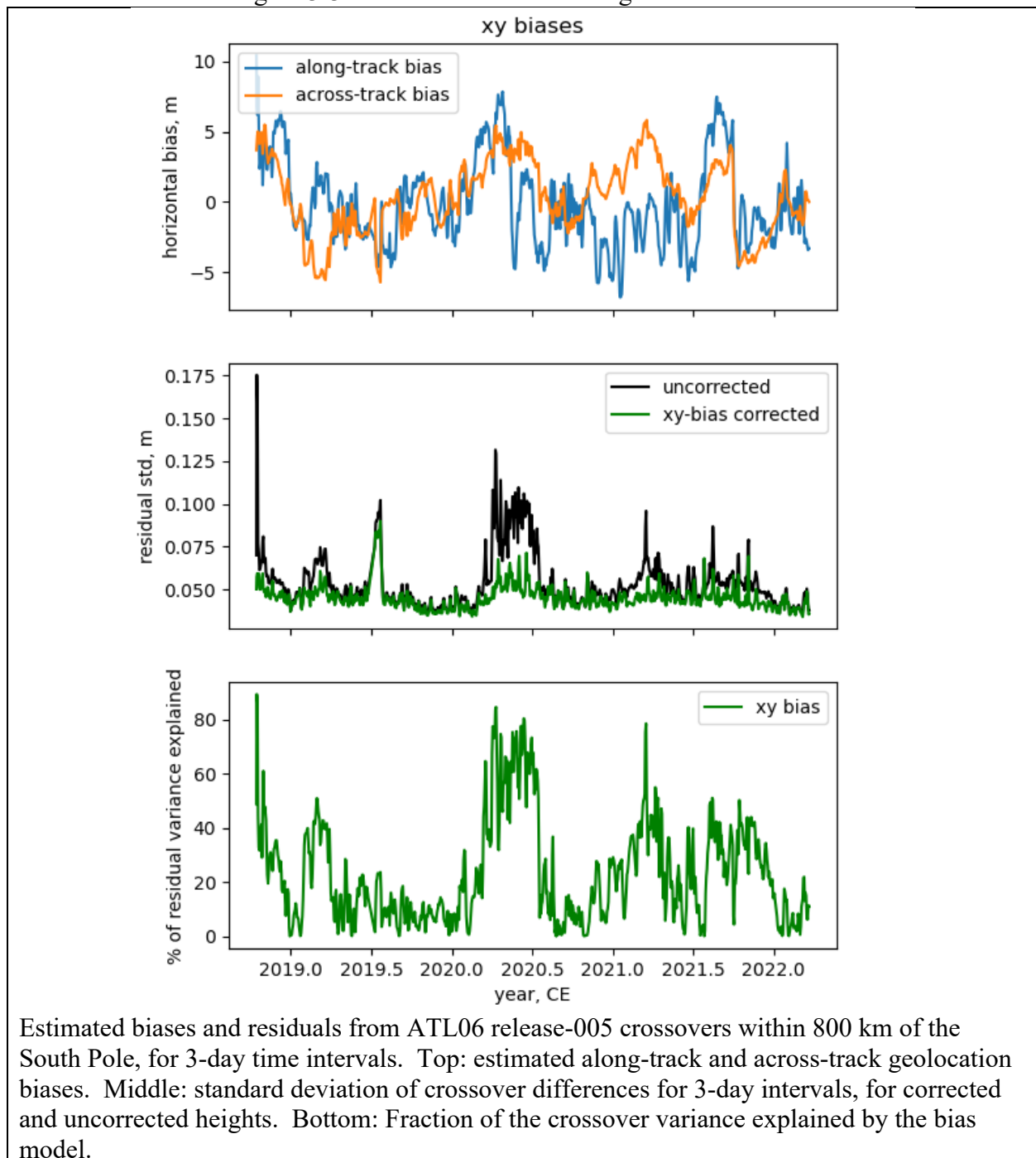
Here we have ignored variations in apparent surface height due to temporal changes in the satellite and surface heights and the geolocation error between the two measurements. We can estimate the bias for a collection of crossover measurements by finding the values of $\delta x_{spot}, \delta y_{spot}$ that minimize the quantity:

$$\sum_{i,j \in \text{crossovers}} \left[\left(h_j - h_i \right) - \left(\frac{x_{spot,j} - x_{spot,i}}{H_{sat} - h_{surf}} \right) \delta x_{spot} + \left(\frac{y_{spot,j} - y_{spot,i}}{H_{sat} - h_{surf}} \right) \delta y_{spot} \right]^2 \quad 20$$

We derive a time series of $\delta x_{spot}, \delta y_{spot}$ by minimizing [] for collections of crossovers chosen so that both measurements in the crossover fall within short (3-day) intervals, spaced regularly (every 1.5 days) throughout the mission. We apply an iterated three-sigma edit as part of the fitting process to avoid letting outlying values in the height-difference distribution influence the recovered value.

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Figure 3-3. Correction for Antarctic geolocation biases



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Figure 3.3 shows the estimated x and y biases for Antarctic crossovers measured for the interior of the ice sheet (within 800 km of the Antarctic pole hole) for cycles 1-15 of the mission, as well time series of RMS crossover differences as a function of time, both before and after the correction is applied, and the fraction of crossover-difference variance explained by the

correction. Over most of the mission, the biases vary between -5 m and +5 m in each direction, with occasional excursions as large as 7 m. The uncorrected crossovers have standard deviations as large as 0.1 m, most consistently in a 3-4 month period in early 2020. Correcting for the biases reduces the crossover standard deviations to the 0.04-0.05 m range, except for the July-2019 period affected by the observatory safe hold, when the residuals remain large, at 0.07-0.1 m. The correction for the geolocation bias accounts for 10-80% of the total crossover-difference variance over most of the mission.

To demonstrate the structure of the errors related to the geolocation biases, we plot the statistics of corrected crossovers using data collected between 1000 and 1800 km from the south pole, at surface elevations greater than 1800m. This is a set of data distinct from that used to generate the bias estimates and includes high-elevation parts of Antarctica where surface-height changes should be small and surface slopes should be small. In figure 3-4, We plot four time series of RMS crossover differences: crossovers where both measurements come from the same beam pair, for crossovers where one measurement comes from a center pair and the other where the comes from an edge pair, and for crossovers where the measurements come from different edge pairs. These plots show that in the uncorrected data, the RMS differences follow a consistent order, smallest for those with both measurements from the same spot, and, in increasing order, those from the same pair but different spots, differences between the center pair and edge pair, and differences between different edge pairs. This matches the expected relation, where the farther apart two spots are in the array, the larger the difference between their biases should be. Correcting for the geolocation errors using the biases estimated from the data close to the polehole (plotted in figure 3.3) yields crossover differences with approximately equal RMS values for all combinations of spots, with typical RMS values between 0.04 and 0.05 m. These statistics suggest that the biases calculated using the polehole data are representative of biases over the rest of Antarctica, and that correcting for these biases makes a significant improvement in height-estimate accuracy, particularly for the edge beam pairs.

3.9.2 Correcting for Antarctic geolocation biases.

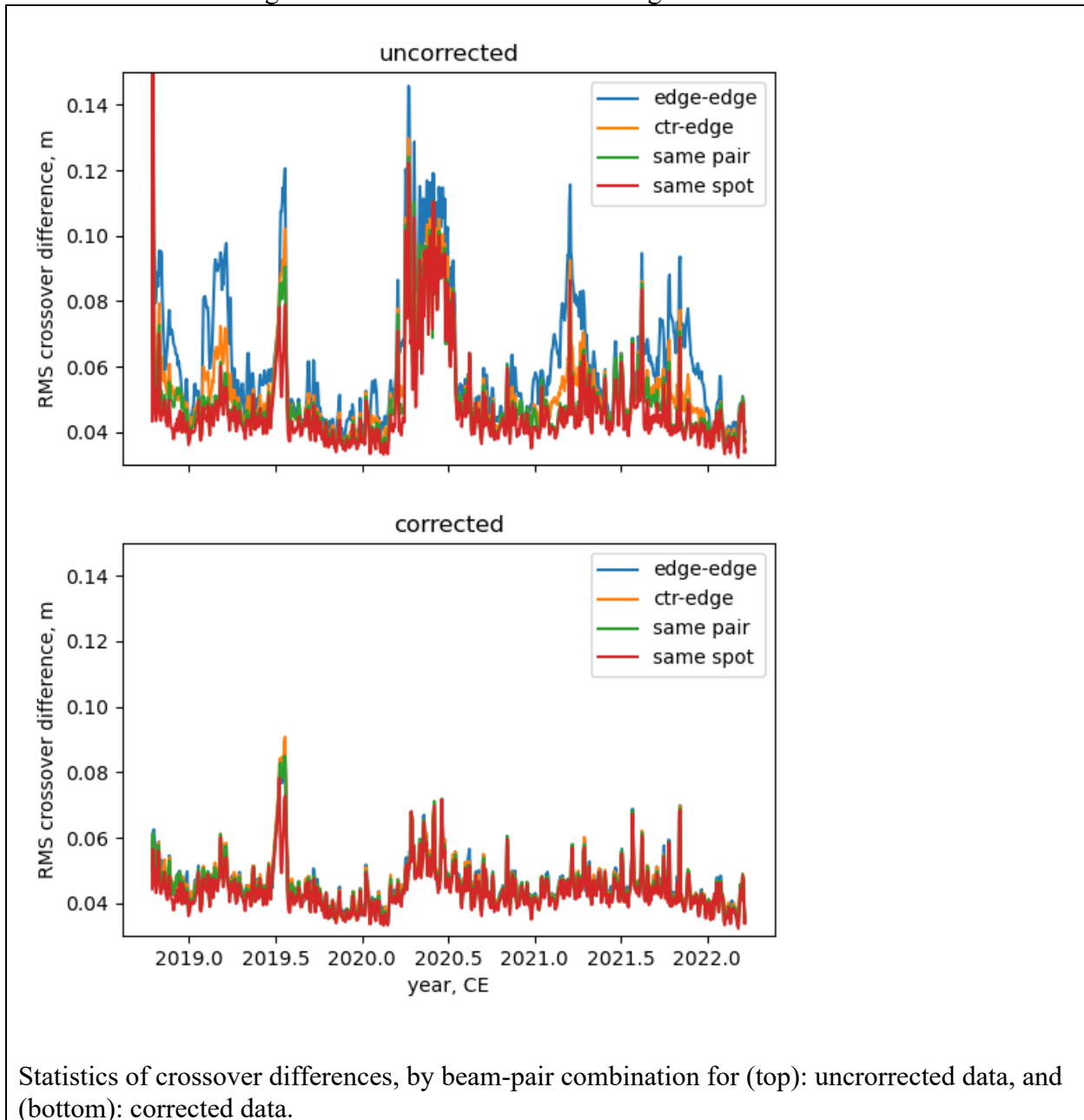
As part of routine data operations for ATL06 production, we generate crossovers for Antarctica. We generate a new set of xy-bias estimates each time a new set of ATL06 data are delivered from SIPS, which are stored as a comma-separated-value file, called, for example:

xy_geoloc_bias_est_AA_800km_3day_20221208.csv

Here AA_800km indicates that the crossovers were generated for Antarctica, for data within 800 km of the polehole, _3day_ indicates the temporal resolution of the time series, and 20221208 indicates that the file was generated on December 12 2022.

We correct for the errors in the spot geolocation by calculating a value of δx_{spot} , δy_{spot} for each ATL06 measurement by linear interpolation into the time series stored in the csv file. We then use equation 18 to calculate the height error for each measurement and subtract it from that measurement's h_{li} value, and adjust the x_{atc} , y_{atc} , values by adding the interpolated δx_{spot} , δy_{spot} values to each. We do not make adjustments to the geolocation values (i.e. latitude and longitude) in the data, because the most significant effect of the geolocation for the

Figure 3-4. Correction for Antarctic geolocation biases



points affects the relative position of the measurements, which is accounted for by shifting the points in along-track coordinates. The mean vertical geolocation correction for each cycle, pair and reference point is stored in the `/ptx/cycle_stats/dh_geoloc` parameter.

4.0 LAND ICE PRODUCTS: LAND ICE H (T)(ATL 11/L3B)

Each ATL11 file contains data for a single reference ground track, for one of the subregions defined for ATLAS granules (see Figure 6-3). The ATL11 consists of three top-level groups, one for each beam pair (*pt1*, *pt2*, *pt3*). Within each pair-track group, there are datasets that give the corrected heights for each cycle, their errors, and the reference-point locations. Subgroups (*cycle_stats*, and *ref_surf*) provide a set of data-quality parameters, and ancillary data describing the fitting process, and use the same ordering and coordinates as the top-level group (i.e. any dataset within the */ptx/cycle_stats* and */ptx/ref_surf* groups refers to the same latitude, longitude, and reference points as the corresponding measurements in the */ptx/* groups.)

Starting in release 007, data for crossover locations is provided in the ATL11xo sub product. Data in this product describes reference-surface-corrected heights from ICESat-2 tracks that cross the reference tracks. These groups provide heights even when ICESat-2 was not following the RGTs, and is the main source on ATL11 for data from cycles 01 and 02, as well as cycles during fill-in campaigns. As described in section 4.5, the ATL11xo product is provided in a tiled format, with one file for 200x200-km subsets of each repeat cycle.

4.1 File naming convention

ATL11 files are named in the following format:

ATL11_ *ttttggccccrrr* _*vv*.h5

Here *tttt* is the rgt number, *gg* is the granule-region number, *cccc* gives the first and last cycles of along-track data included in the file (e.g. *_0308_* would indicate that cycles three through eight, inclusive, might be included in the along-track solution), and *rrr* is the release number. and *vv* is the version number, which is set to one the first time a granule is generated for a given data release, and is incremented by one if the granule is regenerated.

4.2 /ptx group

Table 4-1 shows the datasets in the *ptx* groups. This group gives the principal output parameters of the ATL11. The corrected repeat measurements are in */ptx/h_corr*, which gives improved height measurements based on a surface fit to valid data at paired segments. The associated reference coordinates, */ptx/latitude* and */ptx/longitude* give the reference point location, with averaged times per repeat, */ptx/delta_time*. For repeats with no selected pairs, the corrected height is that from the selected segment with the lowest error. Two error metrics are given in */ptx/h_corr_sigma* and */ptx/h_corr_sigma_systematic*. The first gives the error component due to ATL06 range errors and due to uncertainty in the reference surface. The second gives the component due to geolocation and radial-orbit errors that are correlated at scales larger than one reference point; adding these values in quadrature gives the total per-cycle error. Values are only reported for */ptx/h_corr*, */ptx/h_corr_sigma*, and */ptx/h_corr_sigma_systematic* for those cycles whose uncorrelated errors are less than 15 m; all others are reported as *invalid*. A */ptx/quality_summary* is included for each cycle, based on fit statistics from ATL06.

Table 4-1 Parameters in the /ptx/ group

Parameter	Units	Dimensions	Description
<i>cycle_number</i>	counts	$I \times N_{cycles}$	Cycle number for each column of the data
<i>latitude</i>	degrees North	$N_{pts} \times I$	Reference point latitude
<i>longitude</i>	degrees East	$N_{pts} \times I$	Reference point longitude
<i>ref_pt</i>	counts	$N_{pts} \times I$	The reference point number, m , counted from the equator crossing of the RGT.
<i>delta_time</i>	seconds	$N_{pts} \times N_{cycles}$	mean GPS time for the segments for each cycle, in seconds after the ICESat-2 reference epoch (1 January, 2018)
<i>h_corr</i>	meters	$N_{pts} \times N_{cycles}$	the mean corrected height
<i>h_corr_sigma</i>	meters	$N_{pts} \times N_{cycles}$	the formal error in the corrected height
<i>h_corr_sigma_systematic</i>	meters	$N_{pts} \times N_{cycles}$	the magnitude of the RSS of all errors that might be correlated at scales larger than a single reference point (e.g. pointing errors, GPS errors, etc)
<i>quality_summary</i>	counts	$N_{pts} \times N_{cycles}$	summary flag: zero indicates high-quality cycles: where $\min(\text{signal_selection_source}) \leq 1$ and $\min(\text{SNR_significance}) < 0.02$, and $\text{ATL06_summary_zero_count} > 0$.

836 **4.3 /ptx/ref_surf group**

837 Table 4-2 describes the /ptx/ref_surf group. This group includes parameters describing the
 838 reference surface fit at each reference point. The polynomial coefficients are given in
 839 /ptx/poly_ref_surf, sorted first by total degree, then by x-component degree. Because the
 840 polynomial degree is chosen separately for each reference point, enough columns are provided in
 841 the /ptx/poly_ref_surf and /ptx/poly_ref_surf_sigma to accommodate all possible components up
 842 to 2rd degree in y and 3th degree in x, and absent values are filled in with zeros. The
 843 correspondence between the columns of the polynomial fields and the exponents of the x and y
 844 terms are given in the /ptx/poly_exponent_x and /ptx/poly_exponent_y fields.

Table 4-2 Parameters in the /ptx/ref_surf group

Parameter	Units	Dimensions	Description
dem_h	Meters	$N_{pts} \times 1$	DEM elevation, derived from the ATL06 dem h parameter
geoid_h	Meters	$N_{pts} \times 1$	Geoid height above WGS-84 reference ellipsoid in the tide-free system, derived from ATL06 /gtxx/atl06_segments/dem/geoid_h
complex_surface_flag	counts	$N_{pts} \times 1$	0 indicates that normal fitting was attempted, 1 indicates that the signal selection algorithm rejected too many repeats, and only a linear fit was attempted
rms_slope_fit	counts	$N_{pts} \times 1$	the RMS of the slope of the fit polynomial within 50 m of the reference point
e_slope	counts	$N_{pts} \times 1$	the mean East-component slope for the reference surface within 50 m of the reference point
n_slope	counts	$N_{pts} \times 1$	the mean North-component slope for the reference surface within 50 m of the reference point
at_slope	Counts	$N_{pts} \times 1$	Mean along-track component of the slope of the reference surface within 50 m of the reference point
xt_slope		$N_{pts} \times 1$	Mean across-track component of the slope of the reference surface within 50 m of the reference point
deg_x	counts	$N_{pts} \times 1$	Maximum degree of non-zero polynomial components in x
deg_y	counts	$N_{pts} \times 1$	Maximum degree of non-zero polynomial components in y
poly_exponent_x	counts	1×8	Exponents for the x factors in the surface polynomial
poly_exponent_y	counts	1×8	Exponents for the y factors in the surface polynomial

<i>poly_coeffs</i>	counts	$N_{pts} \times 8$	polynomial coefficients (up to degree 3), for polynomial components scaled by 100 m
<i>poly_ref_coeffs_sigma</i>	counts	$N_{pts} \times 8$	formal errors for the polynomial coefficients
<i>ref_pt_number</i>	counts	$N_{pts} \times 1$	Ref point number, counted from the equator crossing along the RGT.
<i>x_atc</i>	meters	$N_{pts} \times 1$	Along-track coordinate of the reference point, measured along the RGT from its first equator crossing.
<i>y_atc</i>	meters	$N_{pts} \times 1$	Across-track coordinate of the reference point, measured along the RGT from its first equator crossing.
<i>rgt_azimuth</i>	degrees	$N_{pts} \times 1$	Reference track azimuth, in degrees east of local north
<i>misfit_chi2r</i>	meters	$N_{pts} \times 1$	misfit chi square, divided by the number of degrees in the solution
<i>misfit_rms</i>	meters	$N_{pts} \times 1$	RMS misfit for the surface-polynomial fit
<i>fit_quality</i>	counts	$N_{pts} \times 1$	Indicates quality of the fit: 0: no problem identified 1: One or more polynomial coefficient errors larger than 10 2: One or more components of the surface slope has magnitude larger than 0.2 3: Conditions 1 and 2 both true.

The slope of the fit surface is given in the *ref_surf/n_slope* and *ref_surf/e_slope* parameters in the local north and east directions; the corresponding slopes in the along-track and across-track directions are given in the *ref_surf/xt_slope* and *ref_surf/yt_slope* parameters. For the along-track points, the surface slope is calculated by evaluating the correction-surface polynomial for a 10-m spaced grid of points extending ± 50 m in x and y around the reference point, and calculating the mean slopes of these points. The calculation is performed in along-track coordinates and then projected onto the local north and east vectors. The *rms_slope_fit* is derived from the same set of points, and is calculated as the RMS of the standard deviations of the slopes calculated from adjacent grid points, in x and y .

4.4 /ptx/cycle_stats group

The /ptx/cycle_stats group gives summary information about the segments present for each reference point. Most parameters are averaged according to equation 14, but for others (e.g. /ptx/signal_selection_flag_best, which is the minimum of the signal selection flags for the cycle) Table 4-3 describes how the summary statistics are derived.

Table 4-3 Parameters in the /ptx/cycle_stats group

Parameter	Units	Dimensions	Description
<i>ATL06_summary_zero_count</i>	counts	$N_{pts} \times N_{cycles}$	Number of segments with <i>atl06_quality_summary</i> =0 (0 indicates the best-quality data)
<i>h_rms_misfit</i>	meters	$N_{pts} \times N_{cycles}$	Weighted-average RMS misfit between PE heights and along-track land-ice segment fit
<i>dh_geoloc</i>	meters	$N_{pts} \times N_{cycles}$	Correction applied to Antarctic heights based on estimates of the geolocation bias.
<i>r_eff</i>	counts	$N_{pts} \times N_{cycles}$	Weighted-average effective, uncorrected reflectance for each cycle.
<i>tide_ocean</i>	meters	$N_{pts} \times N_{cycles}$	Weighted-average ocean tide for each cycle
<i>dac</i>	meters	$N_{pts} \times N_{cycles}$	Dynamic atmosphere correction (mainly the effect of atmospheric pressure on floating-ice elevation).
<i>cloud_flg_atm</i>	counts	$N_{pts} \times N_{cycles}$	Minimum cloud flag from ATL06: Flag indicates confidence that clouds with $OT^* > 0.2$ are present in the lower 3 km of the atmosphere based on ATL09
<i>cloud_flg_asr</i>	counts	$N_{pts} \times N_{cycles}$	Minimum apparent-surface-reflectance - based cloud flag from ATL06: Flag indicates confidence that clouds with $OT > 0.2$ are present in the lower 3 km of the atmosphere based on ATL09
<i>bsnow_h</i>	meters	$N_{pts} \times N_{cycles}$	Weighted-average blowing snow layer height for each cycle
<i>bsnow_conf</i>	counts	$N_{pts} \times N_{cycles}$	Maximum <i>bsnow_conf</i> flag from ATL06: indicates the greatest (among

Parameter	Units	Dimensions	Description
			segments) confidence flag for presence of blowing snow for each cycle
x_atc	meters	$N_{pts} \times N_{cycles}$	weighted average of pair-center RGT y coordinates for each cycle
y_atc	meters	$N_{pts} \times N_{cycles}$	weighted mean of pair-center RGT y coordinates for each cycle
ref_pt		$N_{pts} \times N_{cycles}$	Ref point number, counted from the equator crossing along the RGT.
seg_count	counts	$N_{pts} \times N_{cycles}$	Number of segments marked as valid for each cycle. Equal to 0 for those cycles not included in the reference-surface shape fit.
$min_signal_selection_source$	counts	$N_{pts} \times N_{cycles}$	Minimum of the ATL06 $signal_selection_source$ value (indicates the highest-quality segment in the cycle)
$min_snr_significance$	counts	$N_{pts} \times N_{cycles}$	Minimum of SNR_significance (indicates the quality of the best segment in the cycle)
$sigma_geo_h$	meters	$N_{pts} \times N_{cycles}$	Root-mean-weighted-square-average total vertical geolocation error due to PPD and POD
$sigma_geo_at$	meters	$N_{pts} \times N_{cycles}$	Root-mean-weighted-square-average local-coordinate x horizontal geolocation error for each cycle due to PPD and POD
$sigma_geo_xt$	meters	$N_{pts} \times N_{cycles}$	Root-mean-weighted-square-average local-coordinate y horizontal geolocation error for each cycle due to PPD and POD
h_mean	meters	$N_{pts} \times N_{cycles}$	Weighted-average of surface heights, not including the correction for the reference surface

*OT (optical thickness) is a measure of signal attenuation used in atmospheric calculations. This parameter discussed in ICESat-2 atmospheric products (ATL09)

4.5 ATL11xo subproduct

The ATL11xo subproduct contains two groups, which together describe the surface at points where datum tracks (the tracks treated in the main ATL11 product) and tracks from other RGTs intersect. The root group in each file contains parameters that are common to the datum track and the crossing track, while the *datum_track* and *crossing_track* groups contain parameters that are associated with the datum and crossing tracks. The data in these groups are complimentary, providing times and heights for each set of measurements as well as tidal and dynamic-atmosphere corrections. Note that *datum_track* height fields may be filled with *invalid* values if no valid data were present on the reference track. This may happen if the satellite was not acquiring data over the RGTs (i.e. for cycles 1 and 2) or if clouds or instrumental problems prevented data acquisition. Conversely, if a height value is not present in the *crossing_track* group for a particular cycle and reference point, there will be no entry in the *datum_track* group for that cycle and reference point.

Because the number of crossing-track data points is small for any single RGT, these data are provided in a format different from the along-track ATL11 data. Instead of being provided in one granule for each RGT, all data for each cycle are collected and stored in spatial subsets called *tiles*, each corresponding to a 200x200-km square in projected coordinates. The filename for each tile is based on the lower-left corner for that tile, so to find data for a location with projected coordinates (x_0, y_0) , the user would read the tile with lower-left-hand coordinates $\left(200 \cdot \text{floor}\left(\frac{x_0}{200 \text{ km}}\right), 200 \cdot \text{floor}\left(\frac{y_0}{200 \text{ km}}\right)\right)$. The projections used are as follows:

- For the Arctic (ICESat-2 regions 3, 4, and 5):

Abbreviation:	AR
Name:	WGS84 / NSIDC Sea Ice Polar Stereographic North projection
EPSG code:	3413
Latitude of origin:	70° N
Central meridian:	-45° E

- For the Antarctic (ICESat-2 regions 10, 11, and 12):

Abbreviation:	AA
Name:	WGS84 / Antarctic Polar Stereographic
EPSG code:	3031
Latitude of origin:	71° S
Central meridian:	0° E

- For lower latitudes (ICESat2 regions 1-2, 6-9, 13-14)

Abbreviation:	LL
Name:	WGS84/Pseudo-Mercator
EPSG code:	3857

Note that the first versions of ATL11 release 007 will be produced only for Arctic and Antarctic regions, so the lower latitude granules will not be calculated. Subsequent versions may include lower-latitude granules.

ATL11xo filenames are as follows:

ATL11xo_{HH}_E{e}_N{n}_c{CC}_{RRR}_{VV}.h5

Here fixed-width fields are given in upper case, variable fields are in lower case:

- HH: two-letter code, indicating AR: Arctic, LL: Lower latitude, AA: Antarctic
- e: Variable-length code, indicating the x coordinate of the lower-left corner of the tile, in projected coordinates
- n: Variable-length code, indicating the y coordinate of the lower-left corner of the tile, in projected coordinates
- CC: Two-digit code indicating the cycle number
- RRR: Three-digit code indicating the data release
- VV: Two-digit code indicating the release number.

As an example, ATL11xo_AR_E200_N-2200_c04_007_01.h5 indicates a tile in the arctic (AR), whose lower-left corner is at 200, -2200 km in projected coordinates, containing data from cycle 04, for release 007, version 01.

The *ref_surf_cycles* attribute of the root group of each ATL11xo file specifies the first and last cycle included in the calculation of the reference surface used in generating the file.

4.5.1 ATL11xo root-level group

The root-level group in ATL11xo contains measurement location information derived from the reference-point location on the datum track, as well as selected parameters describing the reference surface and an index field (*xo_index*) defining the order of the points in all groups in this file. The order of the points is determined by sorting the points first by their y coordinate, rounded to 10 km, then by their x coordinate, also rounded to 10 km, then by their *delta_time* value. This ordering keeps geographically proximate points close to one another in the file, which can help some software access the data more efficiently.

Table 4-4 Parameters in the ATL11xo root-level group

Parameter	Units	Dimensions	Description
<i>latitude</i>	degrees	$N_{XO} \times 1$	latitude of the reference point
<i>longitude</i>	degrees	$N_{XO} \times 1$	longitude of the reference point
<i>x</i>	meters	$N_{XO} \times 1$	Projected x coordinate of the reference point
<i>y</i>	meters	$N_{XO} \times 1$	Projected y coordinate of the reference point
<i>dem_h</i>		$N_{XO} \times 1$	DEM height at the reference point
<i>geoid_h</i>	meters	$N_{XO} \times 1$	geoid height at the reference point

<i>n_slope</i>	unitless	$N_{xo} \times 1$	Local north component of the reference surface slope
<i>e_slope</i>	unitless	$N_{xo} \times 1$	Local east component of the reference surface slope
<i>fit_quality</i>	counts	$N_{xo} \times 1$	Reference point fit quality
<i>xo_index</i>	counts	$N_{xo} \times 1$	Index defining the order of ATL11xo points

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939 4.5.2 ATL11xo/crossing_track group

940 The *crossing_track* group (Table 4-5) contains elevations and times from the crossing tracks,
 941 corrected using the reference surface from the datum track. Each set of values gives the data
 942 from a single segment on the crossing track, that was selected as having the minimum error
 943 among all segments on the crossing track within the $2 L_search_XT$ -by- $2 L_search_AT$ window
 944 around the reference point on the datum track. The systematic errors are evaluated based on the
 945 crossing-track segment location, the magnitude of the datum track's reference-surface slope, and
 946 the magnitude of the horizontal geolocation error. This group contains the parameters from the
 947 crossing track needed to uniquely identify the segment: *rgt*, *pair_track*, and *segment_id*. The
 948 cycle number is the same for all points in a cycle, and may be inferred from the root-level
 949 *cycle_number* attribute or from the granule's filename

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Table 4-5 Parameters in the ATL11xo /crossing_track group

Parameter	Units	Dimensions	Description
<i>delta_time</i>	years	$N_{XO} \times 1$	time relative to the ICESat-2 reference epoch
<i>dh_geoloc</i>	meters	$N_{XO} \times 1$	Correction applied to Antarctic heights based on estimates of the geolocation bias.
<i>h_corr</i>	meters	$N_{XO} \times 1$	WGS-84 height, corrected for the ATL11 surface shape
<i>h_corr_sigma</i>	meters	$N_{XO} \times 1$	error in the height estimate
<i>h_corr_sigma_systematic</i>	meters	$N_{XO} \times 1$	systematic error in the height estimate
<i>tide_ocean</i>	meters	$N_{XO} \times 1$	Ocean-tide estimate
<i>dac</i>	meters	$N_{XO} \times 1$	Dynamic atmosphere correction
<i>rgt</i>	counts	$N_{XO} \times 1$	Reference ground track number
<i>pair_track</i>	counts	$N_{XO} \times 1$	Pair track number for the crossing data
<i>segment_id</i>	counts	$N_{XO} \times 1$	Segment ID for the crossing data
<i>spot</i>	counts	$N_{XO} \times 1$	Spot number
<i>atl06_quality_summary</i>	counts	$N_{XO} \times 1$	Quality flag derived from ATL06. 0 indicates no problems detected, 1 indicates potential problems
<i>along_track_rss</i>	meters	$N_{XO} \times 1$	Root sum of the squared differences between the heights of the endpoints for the crossing-track segment and the centers of the previous and next segments

4.5.3 ATL11xo /datum_track group

The ATL11xo *datum_track* group contains data from the datum track for the same cycle as the crossing track data. The data in this group correspond 1:1 to the data in the *crossing_track* and root-level groups, so that reading these groups produces a set of heights that covers both the reference tracks and the crossing tracks, with one value for each reference point that was measured by a crossing track. The *datum_track* group contains all the fields needed to uniquely identify each ATL11 reference point included: *rgt*, *pair_track*, and *ref_pt*.

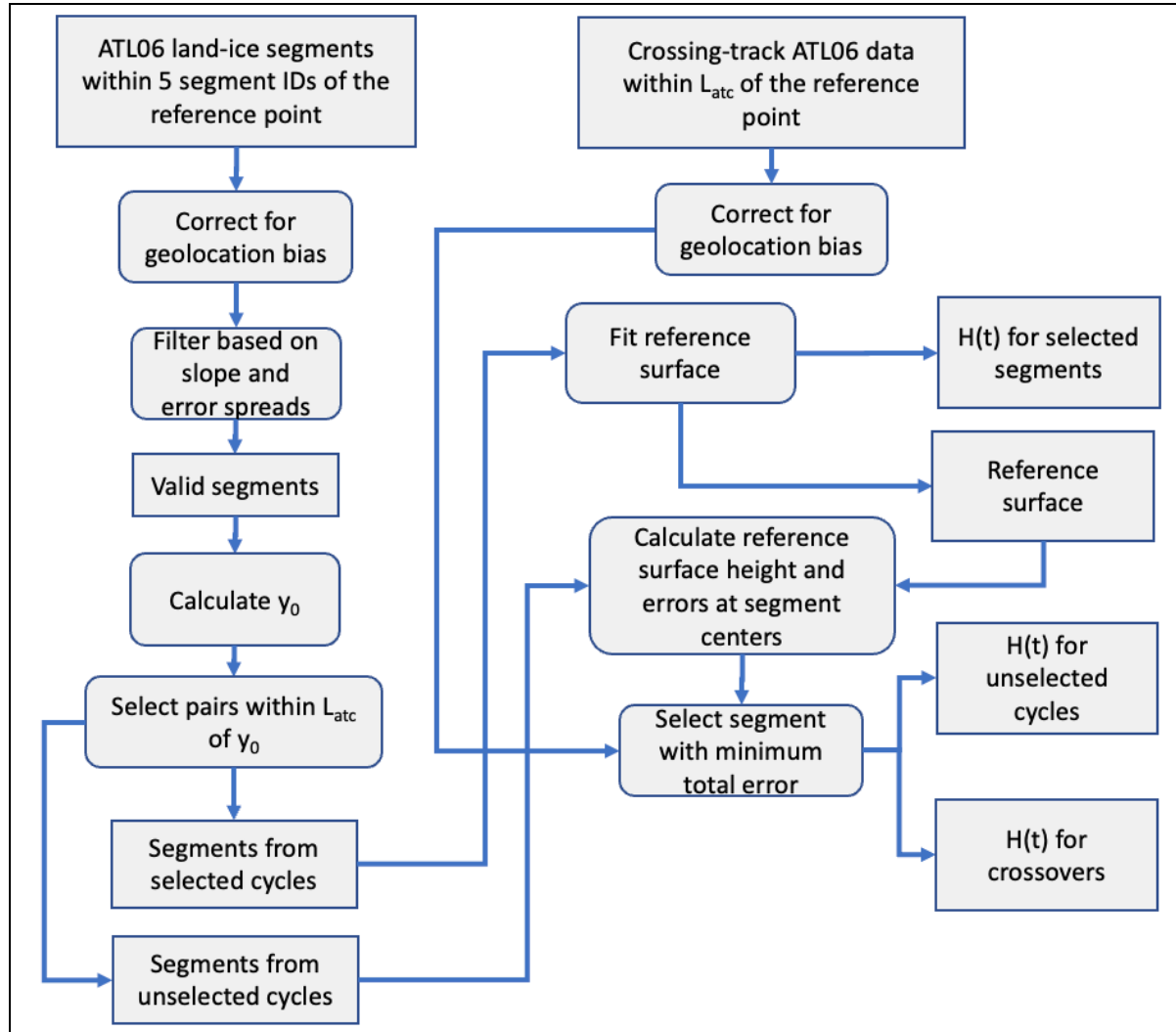
Note that the ATL11xo *datum_track* group contains height measurements that are identical to the corresponding points in the ATL11 (non-crossover) product. These measurements are included solely to simplify within-cycle comparisons between crossing-track and datum-track heights.

Table 4-6 Parameters in the ATL11xo *datum_track* group

Parameter	Units	Dimensions	Description
<i>ref_pt</i>	counts	$N_{XO} \times 1$	the reference-point number for the datum track
<i>rgt</i>	counts	$N_{XO} \times 1$	RGT number for the datum track
<i>pair_track</i>	counts	$N_{XO} \times 1$	Pair track number for the datum track
<i>delta_time</i>	years	$N_{XO} \times 1$	time relative to the ICESat-2 reference epoch
<i>dh_geoloc</i>	meters	$N_{XO} \times 1$	Correction applied to Antarctic heights based on estimates of the geolocation bias
<i>h_corr</i>	meters	$N_{XO} \times 1$	WGS-84 height, corrected for the ATL11 surface shape for the datum track
<i>h_corr_sigma</i>	meters	$N_{XO} \times 1$	error in the height estimate
<i>h_corr_sigma_systematic</i>	meters	$N_{XO} \times 1$	systematic error in the height estimate
<i>tide_ocean</i>	meters	$N_{XO} \times 1$	Ocean-tide estimate
<i>dac</i>	meters	$N_{XO} \times 1$	Dynamic atmosphere correction

5.0 ALGORITHM IMPLEMENTATION

Figure 5-1 Flow Chart for ATL11 Surface-shape Corrections



The following steps are performed for each along-track reference point.

1. Segments with *segment_id* within $N_search/2$ of the reference-point number, are selected.
2. Valid segments are identified based on estimated errors, the *ATL06_quality_summary* parameter, and the along- and across-track segment slopes. Valid pairs, containing valid measurements from two different beams, are also identified.
3. The location of the reference point is adjusted to allow the maximum number of repeats with at least one valid pair to fall within the across-track search distance of the reference point.

4. The reference surface is fit to pairs with two valid measurements within the search distance of the reference point. This calculation also produces corrected heights for the selected pairs and the errors in the correction polynomial coefficients.
 5. The correction surface is used to derive corrected heights for segments not selected in steps 1-3, and the height for the segment with the smallest error is selected for each
 6. The reference surface is used to calculate heights for external (pre-ICESat-2) laser altimetry data sets and crossover ICESat-2 data.
- A schematic of this calculation is shown in Figure 5-1.

5.1.1 Select ATL06 data for the current reference point

Inputs:

ref_pt: segment number for the current reference point
track_num: The track number for current point
pair_num: The pair number for the current point
along_track_shift_table: Estimated geolocation bias lookup table (delta x, y atc vs. delta time)

Outputs:

D_ATL06: ATL06 data structure

Parameters:

N_search: number of segments to search, around *ref_pt*, equal to 5.

Algorithm:

1. For each along-track point, load all ATL06 data from track *track_num* and pair *pair_num* that have *segment_id* within *N_search* of *ref_pt*: These segments have *ref_pt* - *N_search* $\leq$ *segment_id* $\leq$ *ref_pt* + *N_search*.
2. Reject any data that have *y_atc* values more than 500 m distant from the nominal pair-track centers (3200 m for pair 1, 0 m for pair 2, -3200 m for pair 3). Steps 3, 4, and 5 are only carried out for Antarctic data (for Arctic data, *along_track_shift_table* is not specified)
3. Calculate *x_spot* and *y_spot* using equation 17, assuming that the satellite elevation above the ellipsoid is 511 km
4. Interpolate *delta_x_ATC* and *delta_y_ATC* from *along_track_shift_table* as a function of time.
5. Calculate the corrected height using equation 19. Assign the difference between the corrected height and the uncorrected height to the *dh_geoloc* variable in the ATL06 data structure.
6. Subtract *delta_x_ATC* and *delta_y_ATC* from the *x_atc* and *y_atc* variables in the ATL06 data structure.

5.1.2 Select pairs for the reference-surface calculation

Inputs:

ref_pt: reference point number for the current fit
x_atc_ctr: Along-track coordinate of the reference point
D_ATL06: ATL06 data structure
pair_data: Structure describing ATL06 pairs, includes mean of strong/weak beam *y_atc* and *dh_fit_dy*

Outputs:

1023 **validity flags for each segment:**

1024 *valid_segs.x_slope*: Segments identified as valid based on x-slope consistency
 1025 *valid_segs.data*: Segments identified as valid based on ATL06 parameter values.

1026 **Validity flags for each pair:**

1027 *valid_pairs*: Pairs selected for the reference-surface calculation
 1028 *valid_pairs.y_slope*: Pairs identified as valid based on y-slope consistency
 1029 *y_polyfit_ctr*: y center of the slope regression
 1030 *ref_surf/complex_surface_flag*: Flag indicating 0: non-complex surface, 1: complex surface.

1031

1032 **Parameters:**

1033 *L_search_XT*: The across-track search distance.
 1034 *N_search*: Along-track segment search distance
 1035 *seg_sigma_threshold_min*: Minimum threshold for accepting errors in segment heights, equal to
 1036 0.05 m.

1037 **Algorithm:**

1038 1. Flag valid segments based on ATL06 values.

1039 1a. Count the cycles that contain at least one pair that has *atl06_quality_flag*=0
 1040 for both segments. If this number is greater than *N_cycles*/3, set
 1041 *ref_surf/complex_surface_flag*=0 and set *valid_segs.data* to 1 for segments with
 1042 *ATL06_quality_summary* equal to 0. Otherwise, set *ref_surf/complex_surface_flag*=1 and set
 1043 *valid_segs.data* to 1 for segments with *snr_significance* < 0.02.

1044 1b. Define *seg_sigma_threshold* as the maximum of 0.05 or three times the median of
 1045 *sigma_h_li* for segments with *valid_segs.data* equal to 1. Set *valid_segs.data* to 1 for segments
 1046 with *h_sigma_li* less than this threshold and *ATL06_quality_summary* equal to 0.

1047 1c. Define *valid_pairs.data*: For each pair of segments, set *valid_pairs.data* to 1 when
 1048 both segments are marked as valid in *valid_segs.data*.

1049 2. Calculate representative values for the *x* and *y* coordinate for each pair, and filter by distance.

1050 2a. For each pair containing two defined values, set *pair_data.x* to the segments' *x_atc*
 1051 value, and *pair_data.y* to the mean of the segments' *y_atc* values.

1052 2b. Calculate *y_polyfit_ctr*, equal to the median of *pair_data.y* for pairs marked valid in
 1053 *valid_pairs.data*.

1054 2c. Set *valid_pairs.ysearch* to 1 for pairs with $|pair_data.y - y_polyfit_ctr| <$

1055 *L_search_XT*.

1056 3. Select pairs based on across-track slope consistency

1057 3a. Define *pairs_valid_for_y_fit*, for the across-track slope regression if they are marked
 1058 as valid in *valid_pairs.data*, and *valid_pairs.ysearch*, not otherwise.

1059 3b. Choose the degree of the regression for across-track slope

1060 -If the valid pairs contain at least two different *x_atc* values (separated by at least
 1061 18 m), set the along-track degree, *my_regression_y_degree*, to 1, 0 otherwise.

1062 -If valid pairs contain at least two different *ref_surf/y_atc* values (separated by at
 1063 least 18 m), set the across-track degree, *my_regression_y_degree*, to 1, 0 otherwise.

1064 3c. Calculate the formal error in the *y* slope estimates: *y_slope_sigma* is the RSS of the
 1065 *h_li_sigma* values for the two beams in the pair divided by the difference in their *y_atc*
 1066 values. Based on these, calculate *my_regression_tol*, equal to the maximum of 0.01 or three
 1067 times the median of *y_slope_sigma* for valid pairs (*pairs_valid_for_y_fit*).

1068 3d. Calculate the regression of *dh_fit_dy* against *pair_data.x* and *pair_data.y* for valid
1069 pairs (*pairs_valid_for_y_fit*). The result is *y_slope_model*, which gives the variation of *dh_fit_dy*
1070 as a function of *x_atc* and *y_atc*. Calculate *y_slope_resid*, the residuals between the *dh_fit_dy*
1071 values and *y_slope_model* for all pairs in *pair_data*.

1072 3e. Calculate *y_slope_threshold*, equal to the maximum of *my_regression_tol* and three
1073 times the RDE of *y_slope_resid* for valid pairs.

1074 3f. Mark all pairs with $|y_slope_resid| > y_slope_threshold$ as invalid. Re-establish
1075 *pairs_valid_for_y_fit* (based on *valid_pairs.data*, *valid_pairs.y_slope* and *valid_pairs.ysearch*).
1076 Return to step 3d (allow two iterations total).

1077 3g. After the second repetition of 3d-f, use the model to mark all pairs with
1078 $|y_slope_resid|$ less than *y_slope_threshold* with 1 in *valid_pairs.y_slope*, 0 otherwise.

1079 4. Select segments based on along-track slope consistency for both segments in the pair

1080 4a. Define *pairs_valid_for_x_fit*, valid segments for the along-track slope regression:
1081 segments are valid if they come from pairs marked as valid in *valid_pairs.data* and
1082 *valid_pairs.ysearch*, not otherwise.

1083 4b. Choose the degree of the regression for along-track slope
1084 -If valid segments contain at least two different *x_atc* values, set the along-track
1085 degree, *mx_regression_x_degree*, to 1, 0 otherwise.

1086 -If valid segments contain at least two different *y_atc* values, set the across-track
1087 degree, *mx_regression_y_degree*, to 1, 0 otherwise.

1088 4c. Calculate along-track slope regression tolerance, *mx_regression_tol*, equal to the
1089 maximum of either 0.01 or three times the median of the *dh_fit_dx_sigma* values for the valid
1090 pairs.

1091 4d. Calculate the regression of *dh_fit_dx* against *pair_data.x* and *pair_data.y* for valid
1092 segments (*pairs_valid_for_x_fit*). The result is *x_slope_model*, which gives the variation of
1093 *dh_fit_dx* as a function of *pair_data.x* and *pair_data.y*. Calculate *x_slope_resid*, the residuals
1094 between the *dh_fit_dx* and *x_slope_resid* for all segments for this reference point, *seg_x_center*
1095 and *y_polyfit_ctr*.

1096 4e. Calculate *x_slope_threshold*, equal to the maximum of either *mx_regression_tol* or
1097 three times the RDE of *x_slope_resid* for valid segments.

1098 4f. Mark *valid_segs.x_slope* with $|x_slope_resid| > x_slope_threshold$ as invalid. Re-
1099 establish *valid_pairs.x_slope* when both *valid_segs.x_slope* equal 1. Re-establish
1100 *pairs_valid_for_x_fit*. Return to step 4d (allow two iterations total).

1101 4g. After the second repetition of 4d-f, mark all segments with $|x_slope_resid|$ less than
1102 *x_slope_threshold* with 1 in *seg_valid_xslope*, 0 otherwise. Define *valid_pairs.x_slope* as 1 for
1103 pairs that contain two segments with *valid_segs.x_slope*=1, 0 otherwise.

1104 5. Re-establish *valid_pairs.all*. Set equal to 1 if *valid_pairs.x_slope*, *valid_pairs.y_slope*,
1105 and *valid_pairs.data* are all valid.

1106 5a. Identify *unselected_cycle_segs*, as those *D6.cycles* where *valid_pairs.all* are False.

1107

1108

1109 5.1.3 Adjust the reference-point *y* location to include the maximum number of cycles

1110 **Inputs:**

1111 *D_ATL06*: ATL06 structure for the current reference point.

1112 *valid_pairs*: Pairs selected based on parameter values and along- and across-track slopes.

1113 **Outputs:**

1114 *ref_surf/y_atc*: Adjusted fit-point center *y*.

1115 *valid_pairs*: validity masks for pairs, updated to include those identified as valid based on the spatial search around *y_atc_ctr*.

1117 **Parameters:**

1118 *L_search_XT*: Across-track search length (equal to 110 m)

1119 **Algorithm:**

1120 1. Define *y0* as the median of the unique integer values of the pair center *y_atc* for all valid pairs. Set a range of *y* values, *y0_shifts*, as $\text{round}(y0) \pm 100$ meters in 2-meter increments.

1122 2. For each value of *y0_shifts* (*y0_shift*), set a counter, *selected_seg_cycle_count*, to the number of distinct cycles for which both segments of the pair are contained entirely within the *y* interval [*y0_shift* - *L_search_XT*, *y0_shift* + *L_search_XT*]. Add to this, the number of distinct cycles represented by unpaired segments contained within that interval, weighted by 0.01. The sum is called *score*.

1127 3. Search for an optimal *y*-center value (with the most distinct cycles). Set *y_best* to the value of *y0_shift* that maximizes *score*. If there are multiple *y0_shift* values with the same, maximum *score*, set to the median of the *y0_shift* values with the maximum *score*.

1130 4. Update *valid_pairs* to include all pairs with *y_atc* within $\pm L_search_XT$ from *y_atc_ctr*.

1132 5.1.4 Calculate the reference surface and corrected heights for selected pairs

1133 **Inputs:**

1134 *D_ATL06*: ATL06 structure for the current reference point, containing parameters for each segment:

1136 *x_atc*: along-track coordinate

1137 *y_atc*: across-track coordinate

1138 *delta_t*: time for the segment

1139 *pair_data*: Structure containing information about ATL06 pairs. Must include:

1140 *y_atc*: Pair-center across-track coordinates

1141 *valid_pairs*: Pairs selected based on parameter values and along- and across-track slope.

1142 *x_atc_ctr*: The reference point along-track *x* coordinate (equal to *ref_surf/x_atc*).

1143 *y_atc_ctr*: The reference point along-track *x* coordinate (equal to *ref_surf/y_atc*)

1144 **Outputs:**

1145 *ref_surf/deg_x*: Degree of the reference-surface polynomial in the along-track direction

1146 *ref_surf/deg_y*: Degree of the reference-surface polynomial in the across-track direction

1147 *ref_surf/poly_coeffs*: Polynomial coefficients of the reference-surface fit

1148 *ref_surf/poly_coeffs_sigma*: Formal error in polynomial coefficients of the reference-surface fit

1149 *r_seg*: Segment residuals from the reference-surface model

1150 */ptx/h_corr*: Partially filled-in per-cycle corrected height for cycles used in reference surface

1151 */ptx/h_corr_sigma*: Partially filled-in per-cycle formal error in corrected height for cycles used in
 1152 reference surface
 1153 *ref_surf_cycles*: A list of cycles used in defining the reference surface
 1154 *C_m_surf*: Covariance matrix for the reference-polynomial and surface-change model
 1155 *fit_columns_surf*: Mask identifying which components of the combined reference-polynomial
 1156 and surface-change model were included in the fit.
 1157 *poly_exponent_x*: The x degrees corresponding to the columns of matrix used in fitting the
 1158 reference surface to the data
 1159 *poly_exponent_y*: The y degrees corresponding to the columns of matrix used in fitting the
 1160 reference surface to the data
 1161 *selected_segments*: A set of flags indicating which segments were selected by the iterative
 1162 fitting process.
 1163 Partially filled-n per-cycle ATL11 output variables (see table 4-3) for cycles used in reference
 1164 surface
 1165 **Parameters:**
 1166 *poly_max_degree_AT*: Maximum polynomial degree for the along-track fit, equal to 3.
 1167 *poly_max_degree_XT*: Maximum polynomial degree for the across-track fit, equal to 2.
 1168 *max_fit_iterations*: Maximum number of iterations for surface fitting, with acceptable residuals,
 1169 equal to 20.
 1170 *xy_scale*: The horizontal scaling value used in polynomial fits, equal to 100 m
 1171 *t_scale*: The time scale used in polynomial fits, equal to seconds in 1 year.
 1172 **Algorithm:**
 1173 1. Build the cycle design matrix: **G_zp** is a matrix that has one column for each distinct
 1174 cycle in *selected_pairs* and one row for each segment whose pair is in *selected_pairs*. For each
 1175 segment, the corresponding row of **G_zp** is 1 for the column matching the cycle for that segment
 1176 and zero otherwise.
 1177 2. Select the polynomial degree.
 1178 The degree of the x polynomial, *ref_surf/deg_x*, is:
 1179 $\min(\text{poly_max_degree_AT}, \text{maximum}(\text{number of distinct values of } \text{round}((x_{\text{atc}} - x_{\text{atc_ctr}})/20)$
 1180 $\text{among the selected segments in any one cycle}) - 1)$, and the degree of the y polynomial,
 1181 *ref_surf/deg_y*, is : $\min(\text{poly_max_degree_XT}, \text{number of distinct values of}$
 1182 $\text{round}((\text{pair_data.y_atc} - \text{y_atc_ctr})/20) \text{ among the selected pairs})$
 1183 3. Perform an iterative fit for the reference-surface polynomial.
 1184 3a. Define *degree_list_x* and *degree_list_y*: This array defines the x and y degree of the
 1185 polynomial coefficients in the polynomial surface model. There is one component for each
 1186 unique degree combination of x degrees between 0 and *ref_surf/deg_x* and for y degree between
 1187 0 and *ref_surf/deg_y* such that $x_degree + y_degree \leq \max(\text{ref_surf/deg_x}, \text{ref_surf/deg_y})$,
 1188 except that there is no $x_degree=0$ and $y_degree=0$ combination. They are sorted first by the
 1189 sum of the x and y degrees, then by x degree, then by y degree.
 1190 3b. Define the polynomial fit matrix. **S_fit_poly** has one column for each element of
 1191 the polynomial degree arrays, with values equal to $((x_{\text{atc}} - x_{\text{atc_ctr}})/xy_scale)^{x_degree} ((y_{\text{atc}} -$
 1192 $y_{\text{atc_ctr}})/xy_scale)^{y_degree}$. There is one row in the matrix for every segment marked as *selected*.
 1193 3d. Build the surface matrix, **G_surf**, and the combined surface and cycle-height matrix,
 1194 **G_surf_zp**: The surface matrix is equal to **S_fit_poly**. The combined surface and cycle-height
 1195 matrix, **G_surf_zp**, is equal to the horizontal catenation of **G_surf** and **G_zp**.

1196 3e. Subset the fitting matrix. Subset **G_surf_zp** by row to include only rows
1197 corresponding to selected segments to produce **G** (on the first iteration, all are *selected*). Next,
1198 subset **G** by column, first to eliminate all-zero columns, and second to include only columns that
1199 are linearly independent from one another: calculate the normalized correlation between each
1200 pair of columns in **G**, and if the correlation is equal to unity, eliminate the column with the
1201 higher weighted degree ($poly_wt_sum = x_degree + 1.1*y_degree$, with the factor of 1.1
1202 chosen to avoid ties). Identify the selected columns in the matrix as *fit_columns*. If more than
1203 three of the original surface-change columns have been eliminated, set the
1204 *ref_surf/complex_surface_flag* to *True*, mark all columns corresponding to polynomial
1205 coefficients of combined x and y degree greater than 1 as *False* in *fit_columns*.

1206 3f. Check whether the inverse problem is under- or even-determined: If the number of
1207 *selected_segments* is less than the number of columns of **G**, eliminate remaining columns of **G** in
1208 descending order of *poly_wt_sum* until the number of columns of **G** is less than the number of
1209 *selected_segments*.

1210 3g. Generate the data-covariance matrix, **C_d**. The data-covariance matrix is a square
1211 matrix whose diagonal elements are the squares of the *h_li_sigma* values for the selected
1212 segments.

1213 3h. Calculate the polynomial fit. Initialize **m_surf_zp**, the reference model, to a vector of
1214 zero values, with one value for each column of **G_surf_zp**. Calculate the generalized inverse
1215 (equation 7), of **G**, **G_g**. If the inversion calculation returns an error, or if any row of **G_g** is all-
1216 zero (indicating some parameters are not linearly independent), report fit failure and return.
1217 Otherwise, multiply **G_g** by the subset of *h_li* corresponding to the selected segment to give **m**,
1218 containing values for the parameters selected in *fit_columns*. Fill in the components of
1219 **m_surf_zp** flagged in *fit_columns* with the values in **m**.

1220 3i. Calculate model residuals for all segments, *r_seg*, equal to $h_li - G_surf_dz * m_surf_zp$.
1221 The subset of *r_seg* corresponding to *selected* segments is *r_fit*.

1222 3j. Calculate the fitting tolerance, *r_tol*, equal to three times the RDE of the
1223 r_fit/h_li_sigma for all *selected* segments. Calculate the reduced chi-squared value for these
1224 residuals, *ref_surf/misfit_chi2*, equal to $r_fit^T C_d^{-1} r_fit$. Calculate the *P* value for the misfit,
1225 equal to one minus the CDF of a chi-squared distribution with *m-n* degrees of freedom for
1226 *ref_surf/misfit_chi2*, where *m* is the number of rows in **G**, and *n* is the number of columns.

1227 3k. If the *P* value is less than 0.025 and fewer than *max_fit_iterations* have taken place,
1228 mark all segments for which $|r_seg/h_li_sigma| < r_tol$ as *selected*, and return to 3e. Otherwise,
1229 continue to 3k.

1230 3l. Propagate the errors. Based on the most recent value of **C_d**, generate a revised data-
1231 covariance matrix, **C_dp**, whose diagonals values are the maximum of $h_li_sigma^2$ and
1232 $RDE(r_fit)^2$. Calculate the model covariance matrix, **C_m** using equation 9. If any of the
1233 diagonal elements of **C_m** are larger than 10^4 , report a fit failure and return. Fill in elements of
1234 **m_surf_zp** that are marked as valid in *fit_columns* with the square roots of the corresponding
1235 diagonal elements of **C_m**. If any of the errors in the polynomial coefficients are larger than 10,
1236 set *ref_surf/fit_quality*=1.

1237 4. Return a list of cycles used in determining the reference surface in *ref_surf_cycles*. These
1238 cycles have columns in **G** that contain a valid pair, and for which the steps 3e and 3j did not
1239 eliminate the degree of freedom. For these cycles, partially fill in the values of */ptx/h_corr* and
1240 */ptx/h_corr_sigma*, from **m** and **m_sigma**. Similarly, fill in values for
1241 */ptx/h_corr_sigma_systematic* (Equation 12) and */ptx/delta_time*, as well as all variables in Table

1242 4-3. Set */ptx/h_corr*, */ptx/h_corr_sigma*, */ptx/h_corr_sigma_systematic* to NaN for those cycles
 1243 that have uncorrelated error estimates greater than 15 m.
 1244 Values from Table 4-2 defining the fitted reference surface are also reported including
 1245 *ref_surf/poly_coeffs* and *ref_surf/poly_coeffs_sigma*.
 1246 Return **C_m_surf**, the portion of **C_m** corresponding to the polynomial components of
 1247 **C_m**. Return *selected_cols_surf*, the subset of *selected_cols* corresponding to the surface
 1248 polynomial.
 1249 Return the reduced chi-square value for the last iteration, *ref_surf/misfit_chi2r*, equal to
 1250 *ref_surf/misfit_chi2/(m-n)*.
 1251

1252 5.1.5 Calculate corrected heights for cycles with no selected pairs.

1253 **Inputs:**

1254 **C_m_surf**: Covariance matrix for the reference-surface model.
 1255 *degree_list_x*, *degree_list_y*: List of x-, y-, degrees for which the reference-surface calculation
 1256 attempted an estimate.
 1257 *selected_cols_surf*: Parameters of the reference-surface for which the inversion returned a value.
 1258 There should be one value for each row/column of **C_m_surf**.
 1259 *x_atc_ctr*, *y_atc_ctr*: Center point for the surface fit (equal to *ref_surf/x_atc*, *ref_surf/y_atc*)
 1260 *selected_segments*: Boolean array indicating segments selected for the reference-surface
 1261 calculation
 1262 *valid_segs.x_slope*: Segments identified as valid based on x-slope consistency
 1263 *valid_segs.data*: Segments identified as valid based on ATL06 parameter values.
 1264 *pair_number*: Pair number for each segment
 1265 *h_li*: Land-ice height for each segment
 1266 *h_li_sigma*: Formal error in *h_li*.
 1267 */ptx/h_corr*: Partially filled-in per-cycle corrected height
 1268 */ptx/h_corr_sigma*: Partially filled-in per-cycle corrected height error
 1269 *ref_surf/poly_coeffs*: Polynomial coefficients from 2-d reference-surface fit
 1270 *ref_surf_cycles*: A list of cycles used in defining the reference surface
 1271 *ref_surf/N_slope*, *ref_surf/E_slope*: slope components of reference surface
 1272 *sigma_geo_r*: Radial component of the geolocation error for the crossing track
 1273 *D_ATL06*: ATL06 data structure
 1274 Partially filled-in per-cycle ATL11 output variables (see table 4-3)

1275 **Outputs:**

1276 */ptx/h_corr*: Per-cycle corrected height
 1277 */ptx/h_corr_sigma*: Per-cycle corrected height error
 1278 *selected_segments*: A set of arrays listing the selected segments for each cycle.
 1279 Per-cycle ATL11 output variables (see table 4-3).

1280 **Algorithm:**

1281 1. Identify the segments marked as valid in *valid_segs.data* and *valid_segs.x_slope* that are not
 1282 members of the cycles in *ref_surf_cycles*. Label these as *non_ref_segments*.
 1283 2. Build **G_other**, a polynomial-fitting matrix for the *non_ref_segments*. **G_other** will include
 1284 only the polynomial components listed in *degree_list_x* and *degree_list_y*. Multiply **G_other** by
 1285 [*ref_surf/poly_coeffs*] to give corrected heights, *z_kc*.

1286 3. Take the subset of **G_other** corresponding to the components in *fit_cols_surf* to make
 1287 **G_other_surf**. Propagate the polynomial surface errors and surface-height errors for
 1288 *non_ref_segments* based on **G_other_surf**, **C_m_surf**, and *h_li_sigma* using equation
 1289 11. These errors are *z_kc_sigma*.
 1290 4. Identify the segments in *non_ref_segments* for each cycle, and, from among these, select the
 1291 one with the smallest *z_kc_sigma*. If, for this cycle, *z_kc_sigma* is less than 15 m, fill in the
 1292 corresponding values of */ptx/h_corr* and */ptx/h_corr_sigma*. For cycles containing no valid
 1293 segments, report invalid data as NaN. Similarly, fill in the variables in Table 4-3, with the value
 1294 from the segment with the smallest *z_kc_sigma*.
 1295

1296 5.1.6 Calculate corrected heights for crossover data points

1297 **Inputs:**

1298 *C_m_surf*: Covariance matrix for the reference surface model.
 1299 *C_m_surf*: Covariance matrix for the reference-surface model.
 1300 *x_atc_ctr*, *y_atc_ctr*: Center point for the surface fit, in along-track coordinates
 1301 *lat_d*, *lon_d*: Latitude and longitude for the adjusted datum reference point (from */ptx/latitude*,
 1302 */ptx/longitude*)
 1303 *PT*: Pair track for the surface fit
 1304 *RGT*: RGT for the surface fit
 1305 *ref_surf/rgt_azimuth*: The azimuth of the RGT, relative to local north
 1306 *lat_c*, *lon_c*: Location for crossover data
 1307 *time_c*: Time for crossover data
 1308 *h_c*: Elevations for crossover data
 1309 *sigma_h_c*: Estimated errors for crossover data
 1310 *along_track_shift_table*: Estimated geolocation bias lookup table (*delta_x_atc*, *delta_y_atc* vs.
 1311 *delta_time*)
 1312

1313 **Outputs:**

1314 *ref_pt*: reference point (for the reference track)
 1315 *pt*: pair track for the crossing-track points
 1316 *crossing_track/rgt*: Reference ground track for the crossing-track point
 1317 *crossing_track/delta_time*: time for the crossing-track point
 1318 *crossing_track/h_corr*: corrected elevation for the crossing-track points
 1319 *crossing_track/h_corr_sigma*: error in the corrected elevation for the crossing-track points
 1320 *crossing_track/h_corr_sigma_systematic*: Error component in the corrected elevation due to
 1321 pointing and orbital errors.
 1322 *crossing_track/along_track_rss*:

1323 **Parameters:**

1324 *L_search_XT*: Across-track search distance

1325 **Algorithm (executed independently for the data from each cycle of the mission):**

1326 Steps 1, 2, and 3 are only carried out for Antarctic data. For Arctic data,
 1327 *along_track_shift_table* is not specified, and the algorithm begins on step
 1328 1. Calculate *x_spot* and *y_spot* using equation 17, assuming that the satellite elevation
 1329 above the ellipsoid is 511 km

1330 2. Interpolate Δx_{ATC} and Δy_{ATC} from *along_track_shift_table* as a function
 1331 of time.
 1332 3. Calculate the corrected height using equation 19. Assign the difference between
 1333 corrected height and uncorrected height to the Δh_{geoloc} variable in the *crossing_track*
 1334 structure.
 1335 4. Project data points into the along-track coordinate system:
 1336 4a: Calculate along-track and across-track vectors:
 1337 $x_{hat} = [\cos(\text{ref_surf_rgt_azimuth}), \sin(\text{ref_surf_rgt_azimuth})]$
 1338 $y_{hat} = [\sin(\text{ref_surf_rgt_azimuth}), -\cos(\text{ref_surf_rgt_azimuth})]$
 1339 4b. Calculate the R_{earth} , the WGS84 radius at lat_d .
 1340 4c: Project the crossover data points into a local projection centered on the fit center:
 1341 $N_d = R_{earth} (\text{lat}_c - \text{lat}_d)$
 1342 $E_d = R_{earth} \cos(\text{lat}_d) (\text{lon}_c - \text{lon}_d)$
 1343 4d: Calculate the x and y coordinates for the data points, relative to the fit-center point:
 1344 $dx_c = \langle x_{hat}, [E_c, N_c] \rangle$
 1345 $dy_c = \langle y_{hat}, [E_c, N_c] \rangle$
 1346 Here $\langle \mathbf{a}, \mathbf{b} \rangle$ is the inner (dot) product of $\mathbf{a}$ and $\mathbf{b}$.
 1347 5. Calculate the fitting matrix using equation 6.
 1348 6. Calculate the errors at each point using the fitting matrix and C_m , using on equation 11.
 1349 7. Select the minimum-error data point and report the values in Table 4-1.
 1350 8. Calculate the systematic error in the corrected height:
 1351 $\text{crossing_track}/h_sigma_sigma_systematic = (\sigma_{geo_r}^2 + (N_d \text{ref_surf}/n_slope)^2 +$
 1352 $(E_d \text{ref_surf}/e_slope)^2)^{1/2}$
 1353 9. Calculate the along-track RSS for the selected segment. For each selected crossing segment
 1354 calculate the endpoint heights (equal to the segment center height plus or minus 20 meters times
 1355 the segment's along-track slope), and calculate the RSS of the differences between these heights
 1356 and the center heights of the previous and subsequent segments. If this RSS difference is greater
 1357 than 10 m for any cycle, do not report any parameters for that segment's cycle.

1358 5.1.7 Provide error-averaged values for selected ATL06 parameters

1359 **Inputs:**

1360 *ATL06 data structure*: ATL06 data to be averaged

1361 *Selected_segments*: A set of arrays listing the selected segments for each cycle.

1362 *Parameter_list*: A list of parameters to be averaged

1363 **Outputs:**

1364 *Parameter averages*: One value for each parameter and each cycle

1365 **Algorithm:**

1366 1. For each cycle, select the values of h_{li_sigma} based on the values within *selected_segments*.
 1367 Calculate a set of weights, w_i , such that the sum of the weights is equal to 1 and each weight is
 1368 proportional to the inverse square of h_{li_sigma} . If only one value is present in
 1369 *selected_segments*, $w_1 = 1$.
 1370 2. For each parameter, multiply the weights for each cycle by the parameter values, report the
 1371 averaged value in *parameter_averages*.

1372 5.1.8 Provide miscellaneous ATL06 parameters

1373 **Inputs:**

1374 *ATL06 data structure:* ATL06 data to be averaged

1375 *Selected_segments:* A set of arrays listing the selected segments for each cycle.

1376 **Outputs:**

1377 Weighted-averaged parameter values, with one value per cycle, filled in with NaN for cycles
1378 with no selected segments

1379 *cycle_stats/h_robust_sprd*

1380 *cycle_stats/h_li_rms_mean*

1381 *cycle_stats/r_eff*

1382 *cycle_stats/tide_ocean*

1383 *cycle_stats/dac*

1384 *cycle_stats/bsnow_h*

1385 *cycle_stats/x_atc*

1386 *cycle_stats/y_atc*

1387 *cycle_stats/sigma_geo_h*

1388 *cycle_stats/sigma_geo_at*

1389 *cycle_stats/sigma_geo_xt*

1390 *cycle_stats/h_mean*

1391 *ref_surf/dem_h*

1392 *ref_surf/geoid_h*

1393 Parameter minimum values, with one value per cycle, filled in NaN for cycles with no selected
1394 segments:

1395 *cycle_stats/cloud_flg_asr*

1396 *cycle_stats/cloud_flg_atm*

1397 *cycle_stats/bsnow_conf*

1398 Other parameters:

1399 *cycle_stats/strong_spot:* The laser beam number for the strong beam in the pair

1400 **Algorithm:**

1401 1. Select the segments for the cycle indicated in *selected_segments* from the
1402 *ATL06_data_structure*.

1403 2: Based on *h_li_sigma*, calculate the segment weights using equation 14.

1404 3.1 For ATL06 parameters *h_robust_sprd*, *h_li_rms*, *r_eff*, *tide_ocean*, *dac*, *bsnow_h*, *x_atc*,

1405 *y_atc*, *sigma_geo_h*, *sigma_geo_at*, *sigma_geo_xt*, and *h_mean* calculate the weighted average

1406 of the parameter based on the segment weights. The output parameter names are the same as the
1407 input parameter names, in the *cycle_stats* group.

1408 3.2 For ATL06 parameters *dem_h* and *geoid_h*, by regression between the measurement
1409 location and the reference point location. The output parameter names are the same as the input
1410 parameter names, in the *ref_surf* group. If *dem_h* is outside the range of valid *h* values (-400 to
1411 +8400 m), set it to *invalid*.

1412 4. For ATL06 parameters *cloud_flg_asr* and *cloud_flg_atm* report the best (minimum) value
1413 from among the selected values. For *bsnow_conf* report the maximum value from among the
1414 selected values.

1415 5. For the *cycle_stats/strong_spot* attribute, report the laser beam number for the strong beam in
1416 the pair.

1417

1418 5.1.9 Characterize the reference surface

1419 **Inputs:**

1420 *poly_coeffs*: Coefficients of the surface polynomial

1421 *poly_coeff_sigma*: Error estimates for the surface polynomial

1422 *degree_list_x, degree_list_y*: exponents of the reference-surface polynomial for which the

1423 reference-surface fit returned a coefficient

1424 *rgt_azimuth*: the azimuth of the reference ground track

1425 **Parameters:**

1426 *poly_max_degree_AT, poly_max_degree_XT*: Maximum polynomial degree allowed in x and y.

1427 **Outputs:**

1428 *ref_surf/n_slope*: the north component of the reference-surface slope

1429 *ref_surf/e_slope*: the east component of the reference-surface slope

1430 *ref_surf/at_slope*: the along-track component of the reference-surface slope

1431 *ref_surf/xt_slope*: the across-track component of the reference-surface slope

1432 *ref_surf/rms_slope_fit*: the rms slope of the reference surface

1433 *ref_surf/poly_ref_surf*: the polynomial reference surface coefficients

1434 *ref_surf/poly_ref_surf_sigma*: error estimates for *ref_surf/poly_ref_surf*

1435 **Procedure:**

1436 1. Calculate the coordinates of a grid of northing and easting offsets around the reference points,
1437 each between -50 m and 50 m in 10-meter increments: dN, dE

1438 2. Translate the coordinates into along and across-track coordinates:

1439 $dx = \cos(rgt_azimuth) * dN + \sin(rgt_azimuth) * dE$

1440 $dy = \sin(rgt_azimuth) * dN - \cos(rgt_azimuth) * dE$

1441 3. Calculate the polynomial surface elevations for the grid points by evaluating the polynomial
1442 surface at *dx* and *dy*: *z_poly*

1443 4. Fit a plane to z_poly as a function of dN and dE . The North coefficient of the plane is
 1444 ref_surf/n_slope , the east component is ref_surf/e_slope , the RMS misfit of the plane is
 1445 ref_surf/rms_slope_fit . If either component of the slope has a magnitude larger than 0.2, add 2 to
 1446 $ref_surf/fit_quality$.

1447 5. Fit a plane to z_poly as a function of dx and dy . The along-track coefficient of the plane is
 1448 ref_surf/at_slope , the across-track component is ref_surf/xt_slope .

1449 6. Generate the polynomial exponents for the output columns. The list of components for
 1450 the output variables has one component for each unique degree combination of x degrees
 1451 between 0 and ref_surf/deg_x and for y degree between 0 and ref_surf/deg_y such that x_degree
 1452 $+ y_degree \leq \max(poly_max_degree_XT, poly_max_degree_AT)$, except that there is no
 1453 $x_degree=0$ and $y_degree=0$ combination. They are sorted first by the sum of the x and y
 1454 degrees, then by x degree, then by y degree.
 1455 Match the polynomial degrees for this reference point's coefficients to these degrees, and write
 1456 each value of $poly_ref_surf$ and $poly_ref_surf_sigma$ into the appropriate position of the output
 1457 array, filling missing values with *invalid*.

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6.0 APPENDIX A: GLOSSARY

This appendix defines terms that are used in ATLAS ATBDs, as derived from a document circulated to the SDT, written by Tom Neumann. Some naming conventions are borrowed from **Spots, Channels and Redundancy Assignments** (ICESat-2-ATSYS-TN-0910) by P. Luers. Some conventions are different than those used by the ATLAS team for the purposes of making the data processing and interpretation simpler.

Spots. The ATLAS instrument creates six spots on the ground, three that are weak and three that are strong, where strong is defined as approximately four times brighter than weak. These designations apply to both the laser-illuminated spots and the instrument fields of view. The spots are numbered as shown in Figure 1. At times, the weak spots are leading (when the direction of travel is in the ATLAS +x direction) and at times the strong spots are leading. However, the spot number does not change based on the orientation of ATLAS. The spots are always numbered with 1L on the far left and 3R on the far right of the pattern. Not: beams, footprints.

Laser pulse (pulse for short). Individual pulses of light emitted from the ATLAS laser are called laser pulses. As the pulse passes through the ATLAS transmit optics, this single pulse is split into 6 individual transmit pulses by the diffractive optical element. The 6 pulses travel to the earth's surface (assuming ATLAS is pointed to the earth's surface). Some attributes of a laser pulse are the wavelength, pulse shape and duration. Not: transmit pulse, laser shot, laser fire.

Laser Beam. The sequential laser pulses emitted from the ATLAS instrument that illuminate spots on the earth's surface are called laser beams. ATLAS generates 6 laser beams. The laser beam numbering convention follows the ATLAS instrument convention with strong beams numbered 1, 3, and 5 and weak beams numbered 2, 4, and 6 as shown in the figures. Not: beamlet.

Transmit Pulse. Individual pulses of light emitted from the ICESat-2 observatory are called transmit pulses. The ATLAS instrument generates 6 transmit pulses of light from a single laser pulse. The transmit pulses generate 6 spots where the laser light illuminates the surface of the earth. Some attributes of a given transmit pulse are the wavelength, the shape, and the energy. Some attributes of the 6 transmit pulses may be different. Not: laser fire, shot, laser shot, laser pulse.

Reflected Pulse. Individual transmit pulses reflected off the surface of the earth and viewed by the ATLAS telescope are called reflected pulses. For a given transmit pulse, there may or may not be a reflected pulse. Not: received pulse, returned pulse.

Photon Event. Some of the energy in a reflected pulse passes through the ATLAS receiver optics and electronics. ATLAS detects and time tags some fraction of the photons that make up the reflected pulse, as well as background photons due to sunlight or instrument noise. Any photon that is time tagged by the ATLAS instrument is called a photon event, regardless of source. Not: received photon, detected photon.

Reference Ground Track (RGT). The reference ground track (RGT) is the track on the earth at which a specified unit vector within the observatory is pointed. Under nominal operating conditions, there will be no data collected along the RGT, as the RGT is spanned by GT2L and GT2R (which are not shown in the figures, but are similar to the GTs that are shown). During spacecraft slews or off pointing, it is possible that ground tracks may intersect the RGT. The precise unit vector has not yet been defined. The ICESat-2 mission has 1387 RGTs, numbered from 0001xx to 1387xx. The last two digits refer to the cycle number. Not: ground tracks, paths, sub-satellite track.

Cycle Number. Over 91 days, each of the 1387 RGTs will be targeted in the Polar Regions once. In subsequent 91-day periods, these RGTs will be targeted again. The cycle number tracks the number of 91-day periods that have elapsed since the ICESat-2 observatory entered the science orbit. The first 91-day cycle is numbered 01; the second 91-day cycle is 02, and so on. At the end of the first 3 years of operations, we expect the cycle number to be 12. The cycle number will be carried in the mid-latitudes, though the same RGTs will (in general) not be targeted more than once.

Sub-satellite Track (SST). The sub-satellite track (SST) is the time-ordered series of latitude and longitude points at the geodetic nadir of the ICESat-2 observatory. In order to protect the ATLAS detectors from damage due to specular returns, and the natural variation of the position of the observatory with respect to the RGT throughout the orbit, the SST is generally not the same as the RGT. Not: reference ground track, ground track.

Ground Tracks (GT). As ICESat-2 orbits the earths, sequential transmit pulses illuminate six ground tracks on the surface of the earth. The track width is approximately 10m wide. Each ground track is numbered, according to the laser spot number that generates a given ground track. Ground tracks are therefore always numbered with 1L on the far left of the spot pattern and 3R on the far right of the spot pattern. Not: tracks, paths, reference ground tracks, footpaths.

Reference Pair Track (RPT). The reference pair track is the imaginary line halfway between the planned locations of the strong and weak ground tracks that make up a pair. There are three RPTs: RPT1 is spanned by GT1L and GT1R, RPT2 is spanned by GT2L and GT2R (and may be coincident with the RGT at times), and RPT3 is spanned by GT3L and GT3R. Note that this is the planned location of the midway point between GTs. We will not know this location very precisely prior to launch. Not: tracks, paths, reference ground tracks, footpaths, pair tracks.

Pair Track (PT). The pair track is the imaginary line half way between the actual locations of the strong and weak ground tracks that make up a pair. There are three PTs: PT1 is spanned by GT1L and GT1R, PT2 is spanned by GT2L and GT2R (and may be coincident with the RGT at times), and PT3 is spanned by GT3L and GT3R. Note that this is the actual location of the midway point between GTs, and will be defined by the actual location of the GTs. Not: tracks, paths, reference ground tracks, footpaths, reference pair tracks.

Pairs. When considered together, individual strong and weak ground tracks form a pair. For example, GT2L and GT2R form the central pair of the array. The pairs are numbered 1 through

3: Pair 1 is comprised of GT1L and GT1R, pair 2 is comprised of GT2L and GT2R, and pair 3 is comprised of GT3L and 3R.

Along-track. The direction of travel of the ICESat-2 observatory in the orbit frame is defined as the along-track coordinate, and is denoted as the +x direction. The positive x direction is therefore along the Earth-Centered Earth-Fixed velocity vector of the observatory. Each pair has a unique coordinate system, with the +x direction aligned with the Reference Pair Tracks.

Across-track. The across-track coordinate is y and is positive to the left, with the origins at the Reference Pair Tracks.

Segment. An along-track span (or aggregation) of PE data from a single ground track or other defined track is called a segment. A segment can be measured as a time duration (e.g. from the time of the first PE to the time of the last PE), as a distance (e.g. the distance between the location of the first and last PEs), or as an accumulation of a desired number of photons. Segments can be as short or as long as desired.

Signal Photon. Any photon event that an algorithm determines to be part of the reflected pulse.

Background Photon. Any photon event that is not classified as a signal photon is classified as a background photon. Background photons could be due to noise in the ATLAS instrument (e.g. stray light, or detector dark counts), sunlight, or mis-classified signal photons. Not: noise photon.

h_{}.** Signal photons will be used by higher-level products to determine height above the WGS-84 reference ellipsoid, using a semi-major axis (equatorial radius) of 6378137m and a flattening of 1/298.257223563. This can be abbreviated as ‘ellipsoidal height’ or ‘height above ellipsoid’. These heights are denoted by h; the subscript ** will refer to the specific algorithm used to determine that elevation (e.g. is = ice sheet algorithm, si = sea ice algorithm, etc...). Not: elevation.

Photon Cloud. The collection of all telemetered photon time tags in a given segment is the (or a) photon cloud. Not: point cloud.

Background Count Rate. The number of background photons in a given time span is the background count rate. Therefore a value of the background count rate requires a segment of PEs and an algorithm to distinguish signal and background photons. Not: Noise rate, background rate.

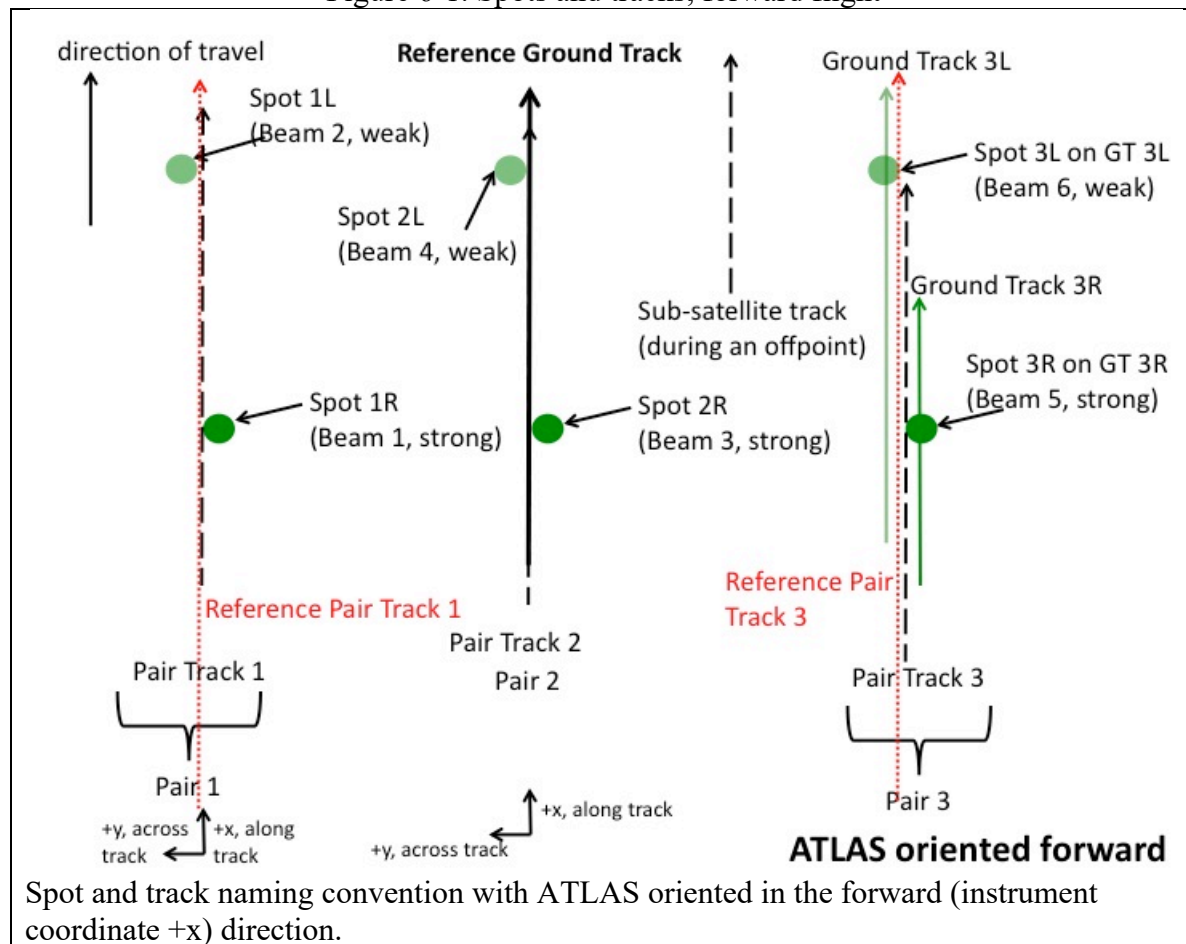
Noise Count Rate. The rate at which the ATLAS instrument receives photons in the absence of any light entering the ATLAS telescope or receiver optics. The noise count rate includes PEs due to detector dark counts or stray light from within the instrument. Not: noise rate, background rate, and background count rate.

Telemetry band. The subset of PEs selected by the science algorithm on board ATLAS to be telemetered to the ground is called the telemetry band. The width of the telemetry band is a

function of the signal to noise ratio of the data (calculated by the science algorithm onboard ATLAS), the location on the earth (e.g. ocean, land, sea ice, etc...), and the roughness of the terrain, among other parameters. The widths of telemetry bands are adjustable on-orbit. The telemetry bandwidth is described in Section 7 or the ATLAS Flight Science Receiver Algorithms document. The total volume of telemetered photon events must meet the data volume constraint (currently 577 GBits/day).

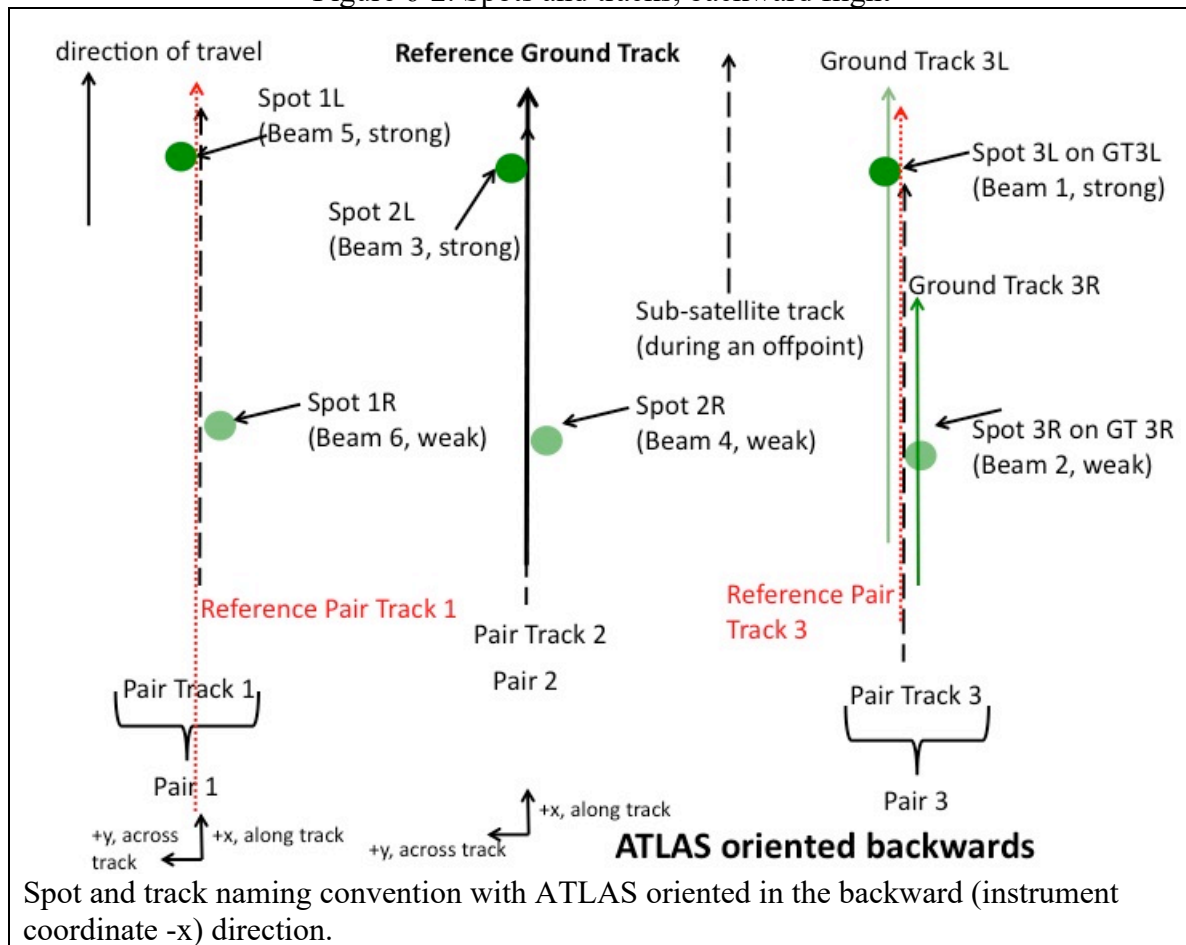
Window, Window Width, Window Duration. A subset of the telemetry band of PEs is called a window. If the vertical extent of a window is defined in terms of distance, the window is said to have a width. If the vertical extent of a window is defined in terms of time, the window is said to have a duration. The window width is always less than or equal to the telemetry band.

Figure 6-1. Spots and tracks, forward flight



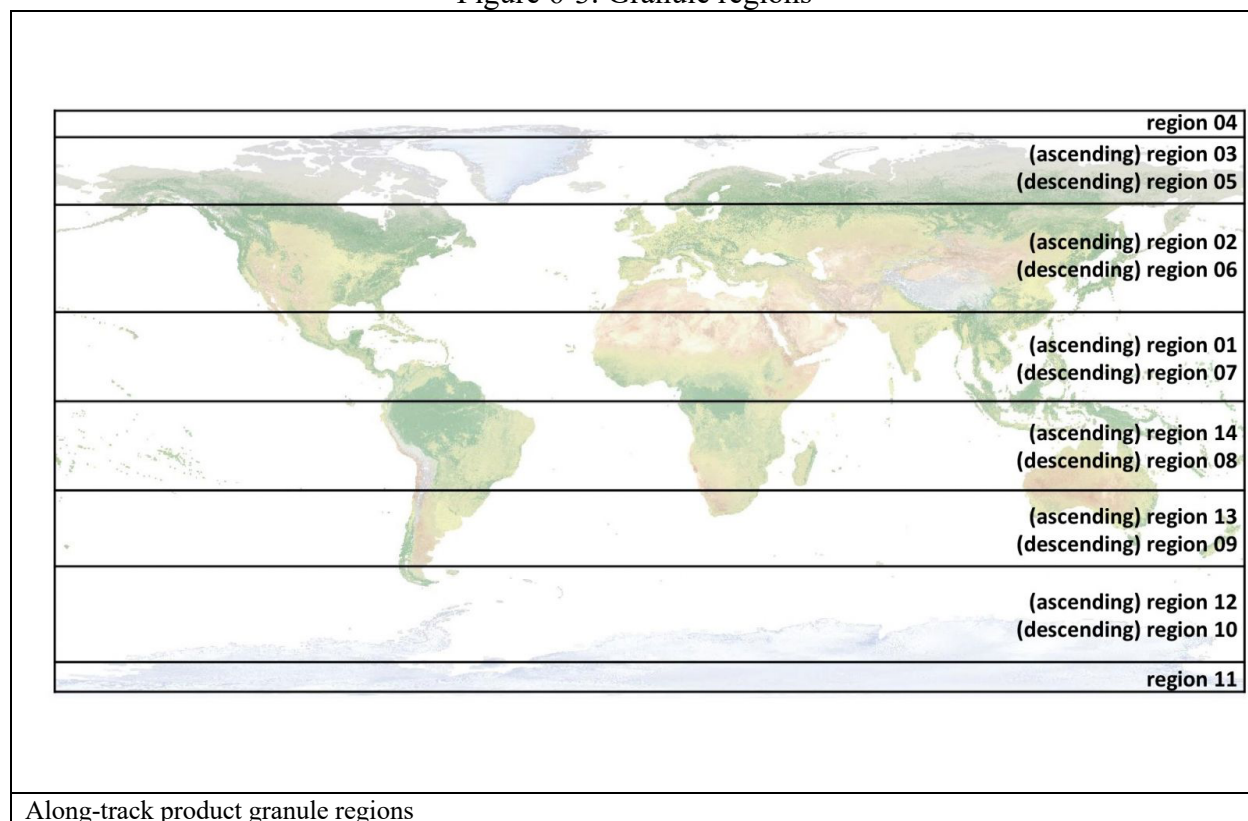
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Figure 6-2. Spots and tracks, backward flight



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Figure 6-3. Granule regions

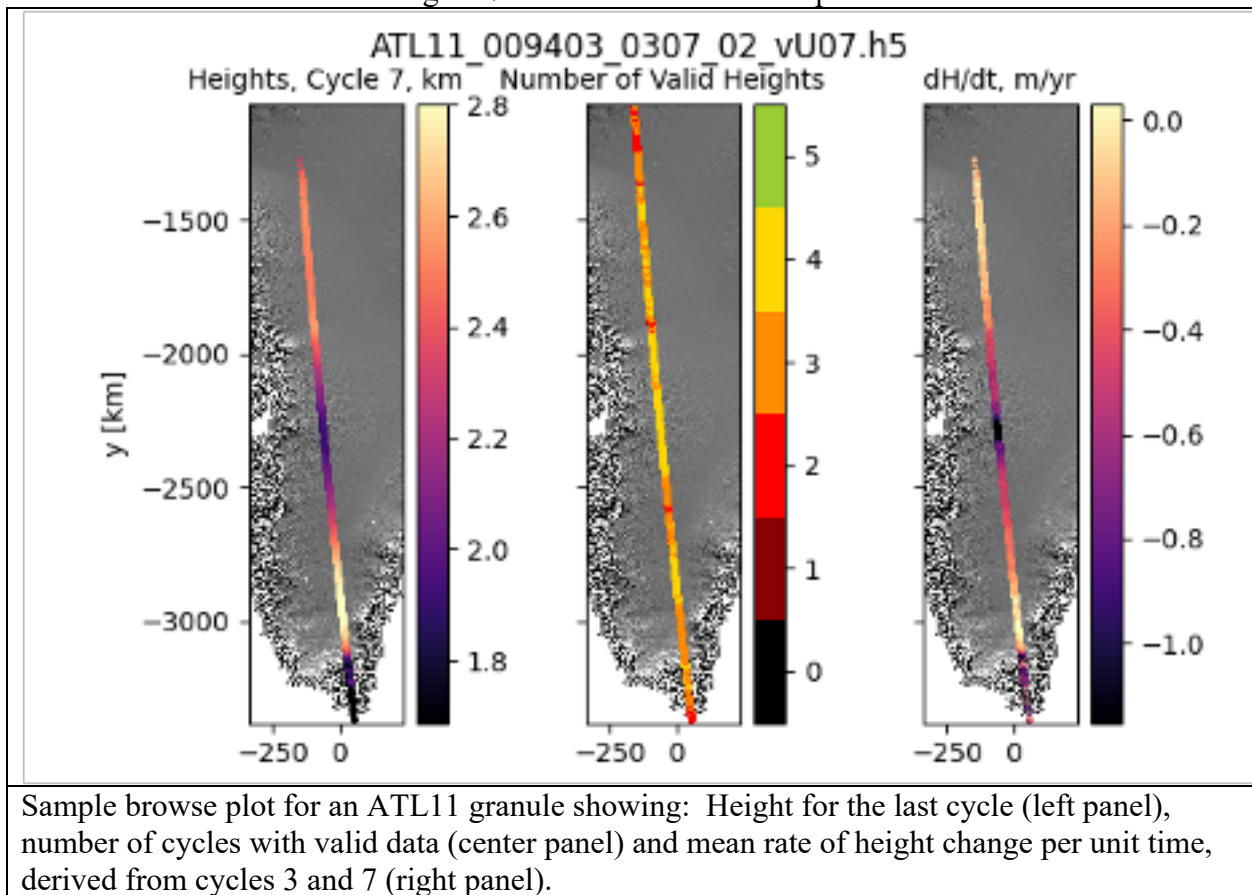


7.0 BROWSE PRODUCTS

For each ATL11 data file, there will be eight figures written to an associated browse file. Two of these figures are required and are located in the default group; default1 and default2. The browse filename has the same pattern as the data filename, namely, ATL11_ttttss_c1c2_rr_vVVV_BRW.h5, where tttt is the reference ground track, ss is the orbital segment, c1 is the first cycle of data in the file, c2 is the last cycle of data in the file, rr is the release and VVV is the version. Optionally, the figures can also be written to a pdf file.

Below is a discussion of the how the figures are made, with examples from the data file ATL11_009403_0307_02_vU07.h5. Note that the figure numbering in this section is distinct from that in the rest of the document; the figures shown here are labeled as they are in each browse-product file.

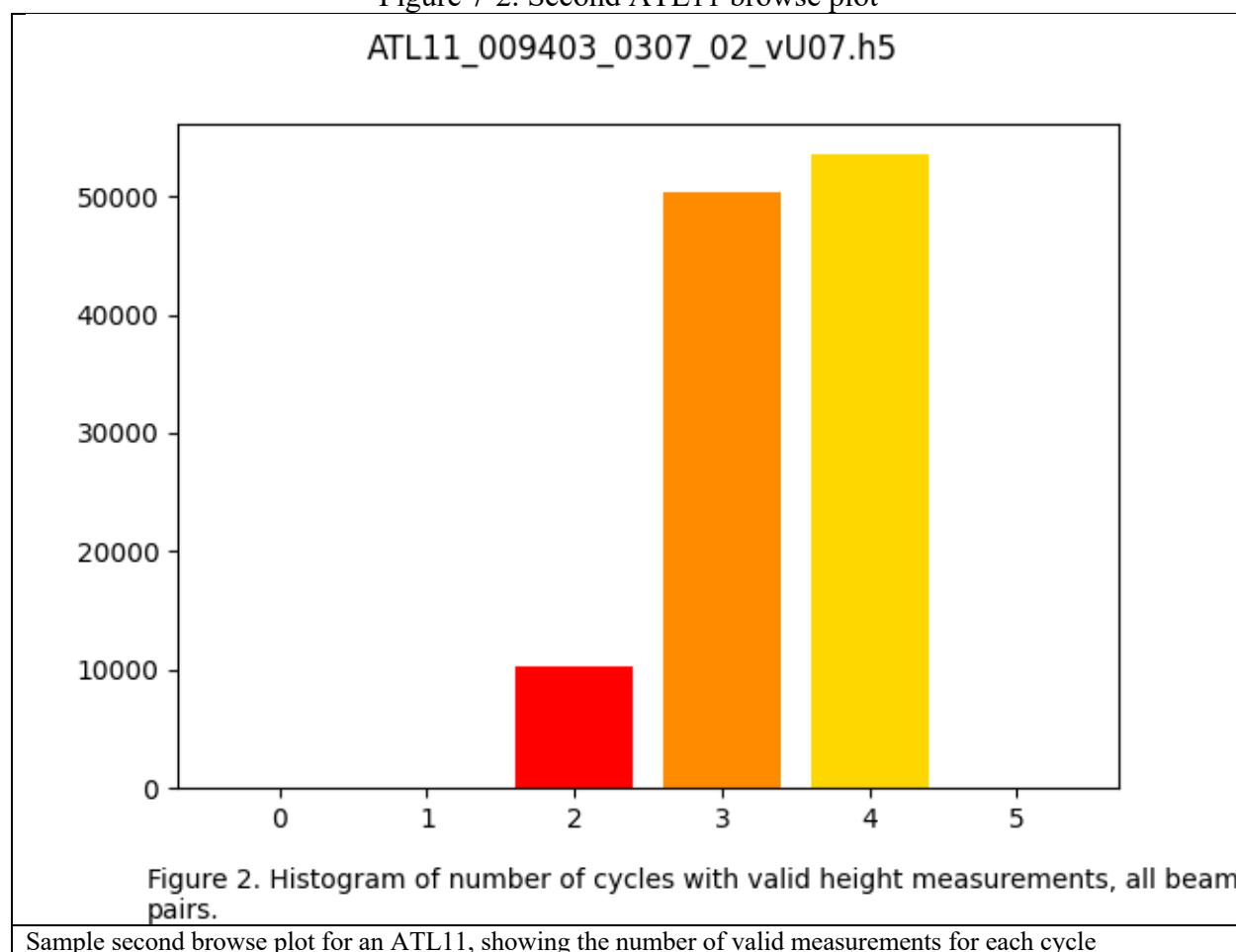
Figure 7-1. First ATL11 browse plot



The background for the three panels in Figure 7-1 is the gradient DEM in gray scale. It is shown in a polar-stereographic projection with a central longitude of 45W (0E) and a standard latitude of 70N (71S), for the Northern (Southern) Hemisphere. The map is bounded by the extent of height data plus a buffer. ATL11 heights (/ptx/h_corr) from all pairs of the latest cycle with valid data, here cycle 7, are plotted in the first panel. The “magma” color map indicates the heights in km. The limits on the color bar are set with the python scipy.stat.scoreatpercentile method at 5%

and 95%. In the second panel are plotted the number of valid heights summed over all cycles at each location. The color bar extends to the total number of cycles in the data file. The change in height over time, dH/dt , is plotted in the third panel, in meters/year. $dHdt$ is the change in height of the last cycle with valid data from the first cycle with valid data ($/ptx/h_corr$) divided by the associated times ($/ptx/delta_time$). Text of 'No Data' is printed in the panel if there is only one cycle with valid data, or if the first and last cycles with valid data have no common reference point numbers ($/ptx/ref_pt$). All plots are in x,y coordinates, in km. This figure is called default/default1 in the BRW.h5 file.

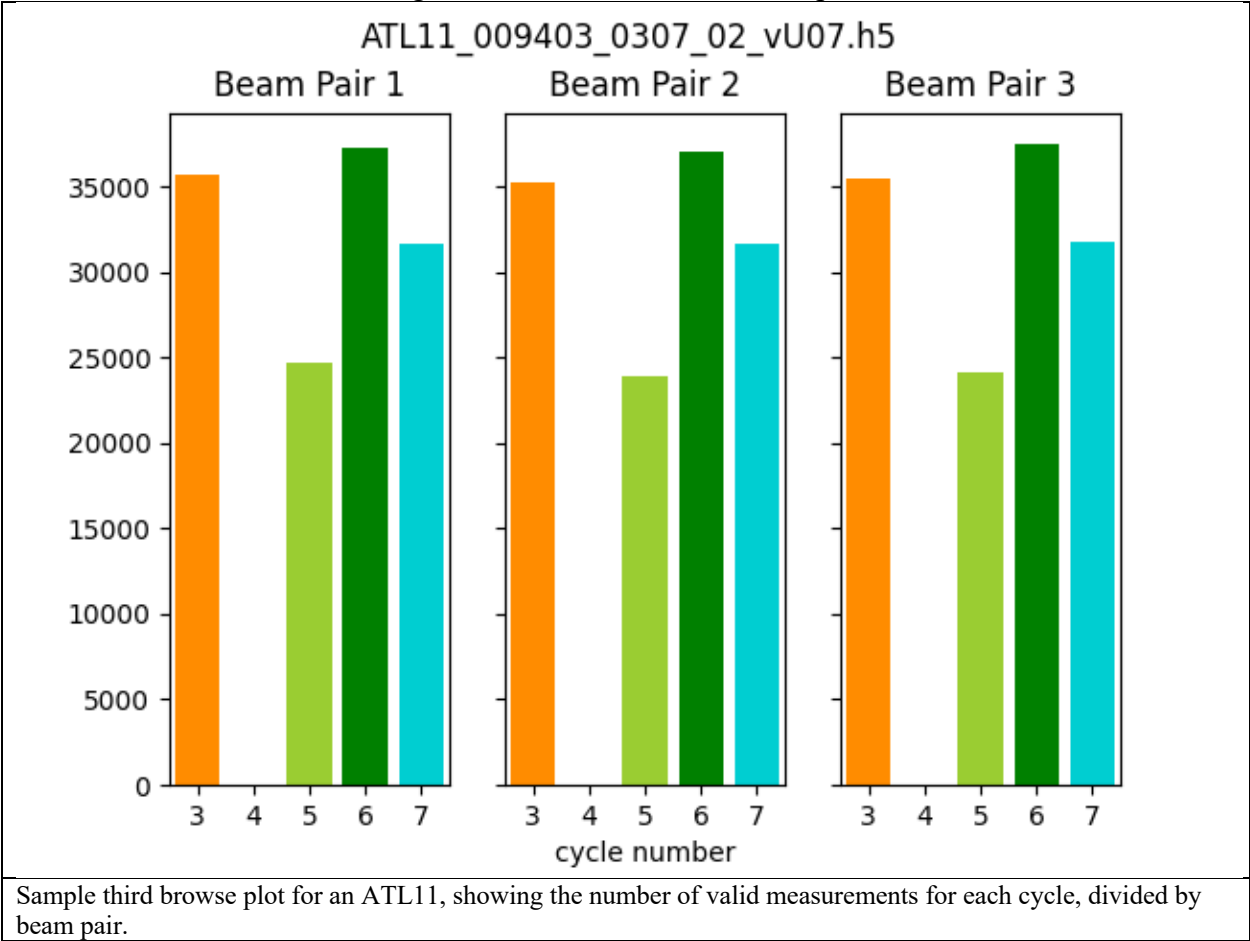
Figure 7-2. Second ATL11 browse plot



A histogram of the number of valid height measurements ($/ptx/h_corr$) is in Figure 2. Valid height data are summed across all cycles, for each reference point number ($/ptx/ref_pt$). The color scale is from zero to the total number of cycles in the data file and matches those in Figure 1, 2nd panel. This figure is called validrepeats_hist in the BRW.h5 file.

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Figure 7-3. Third ATL11 browse plot

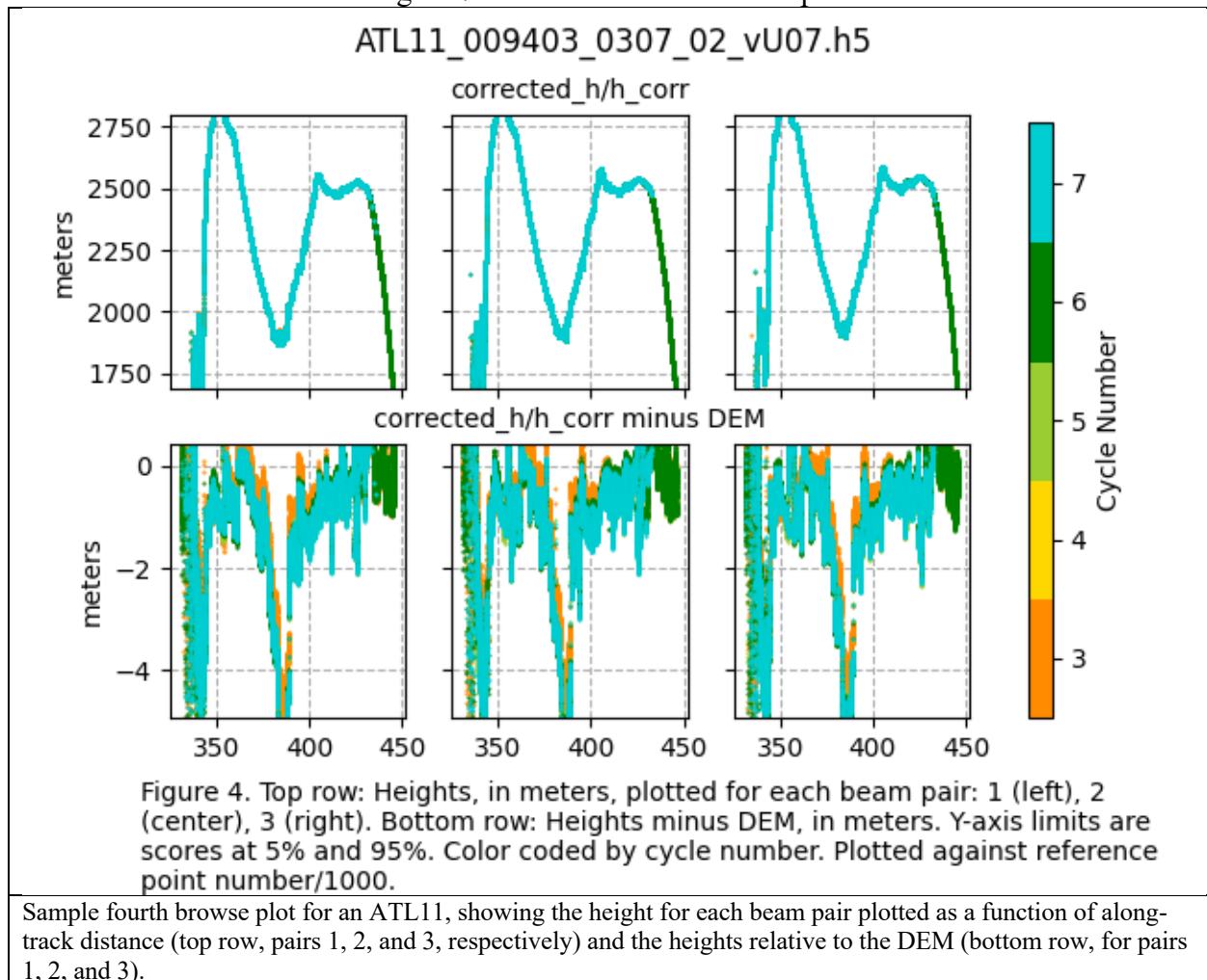


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Histograms in Figure 7-3 show the number of valid heights (/ptx/h_corr) for each cycle, separated by beam pair. The cycle numbers are color coded. This figure is called default/default2 in the BRW.h5 file.

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Figure 7-4. Fourth ATL11 browse plot



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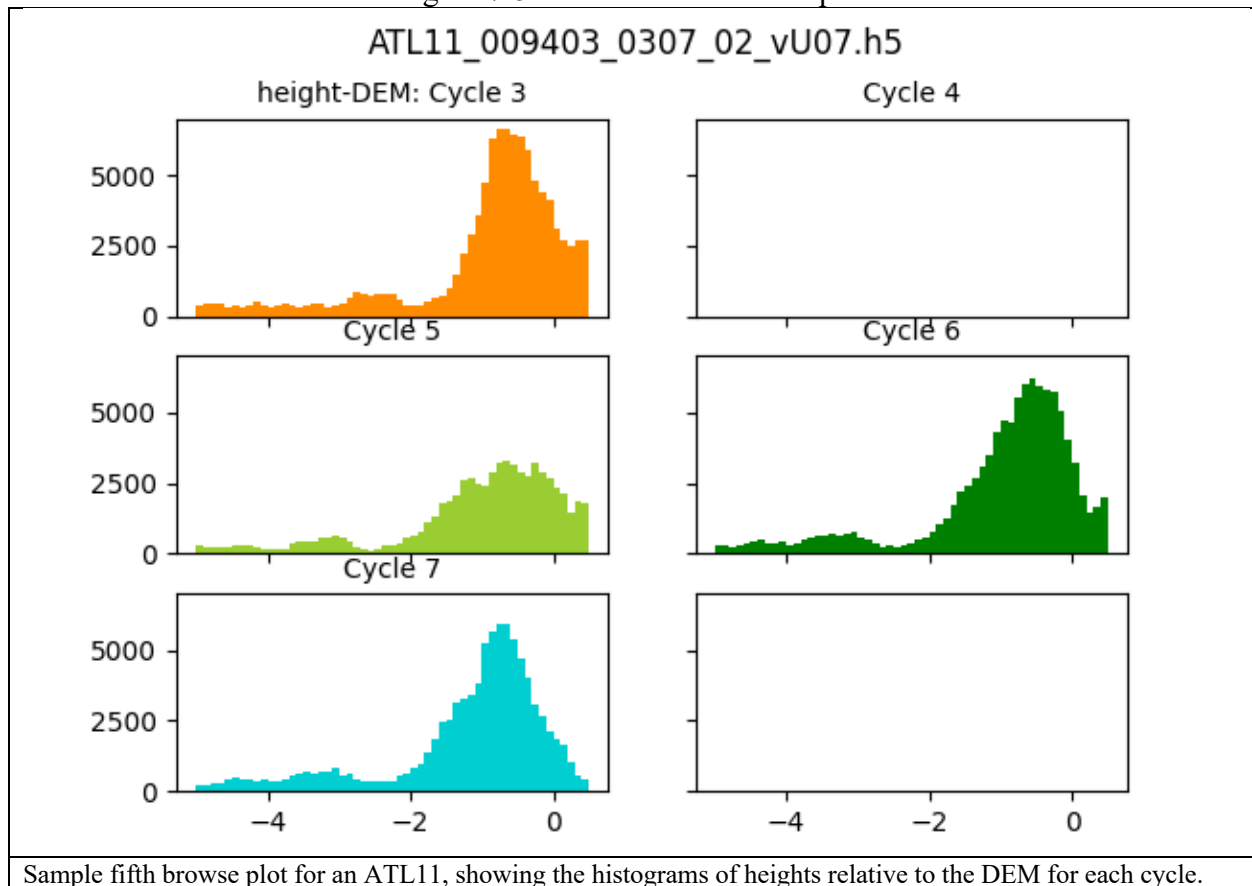
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There are six panels in Figure 7-4, with two rows and three columns. In the top row are plotted the height measurements ($/\text{ptx}/h_corr$) for each beam pair, one pair per panel. In the bottom row are plotted the same height measurements minus the collocated DEM ($\text{ref_surf}/\text{dem_h}$) values, one pair per panel. The plots are color coded by cycle number, as in Figure 3. The heights are plotted versus reference point number ($/\text{ptx}/\text{ref_pt}$) divided by 1000 for a cleaner plot. The y-axis is in meters for both rows. The y-axis limits for the top and bottom rows are set separately, using the python `scipy.stats.scoreatpercentile` method with limits of 5% and 95% for heights and height differences, respectively. Text of 'No Data' is printed in a panel if there are no valid height data for that pair. This figure is called `h_corr_h_corr-DEM` in the `BRW.h5` file.

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Figure 7-5. Fifth ATL11 browse plot



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1692 Figure 7-5 is associated with Figure 7-4. It is a multi-paneled figure, with the number of panels
 1693 dependent on the number of cycles in the data file. Each panel is a histogram of the heights
 1694 ($\text{ptx}/h_{\text{corr}}$) minus collocated DEM heights ($\text{ref_surf}/\text{dem_h}$) color coded by cycle, the same as
 1695 in Figures 3 and 4. The limits on the histograms are set using the python
 1696 `scipy.stats.scoreatpercentile` method with limits of 5 and 95% for all cycles of data, the same
 1697 values used in Figure 4 bottom row. This figure is called `h_corr-DEM_hist` in the `BRW.h5` file.

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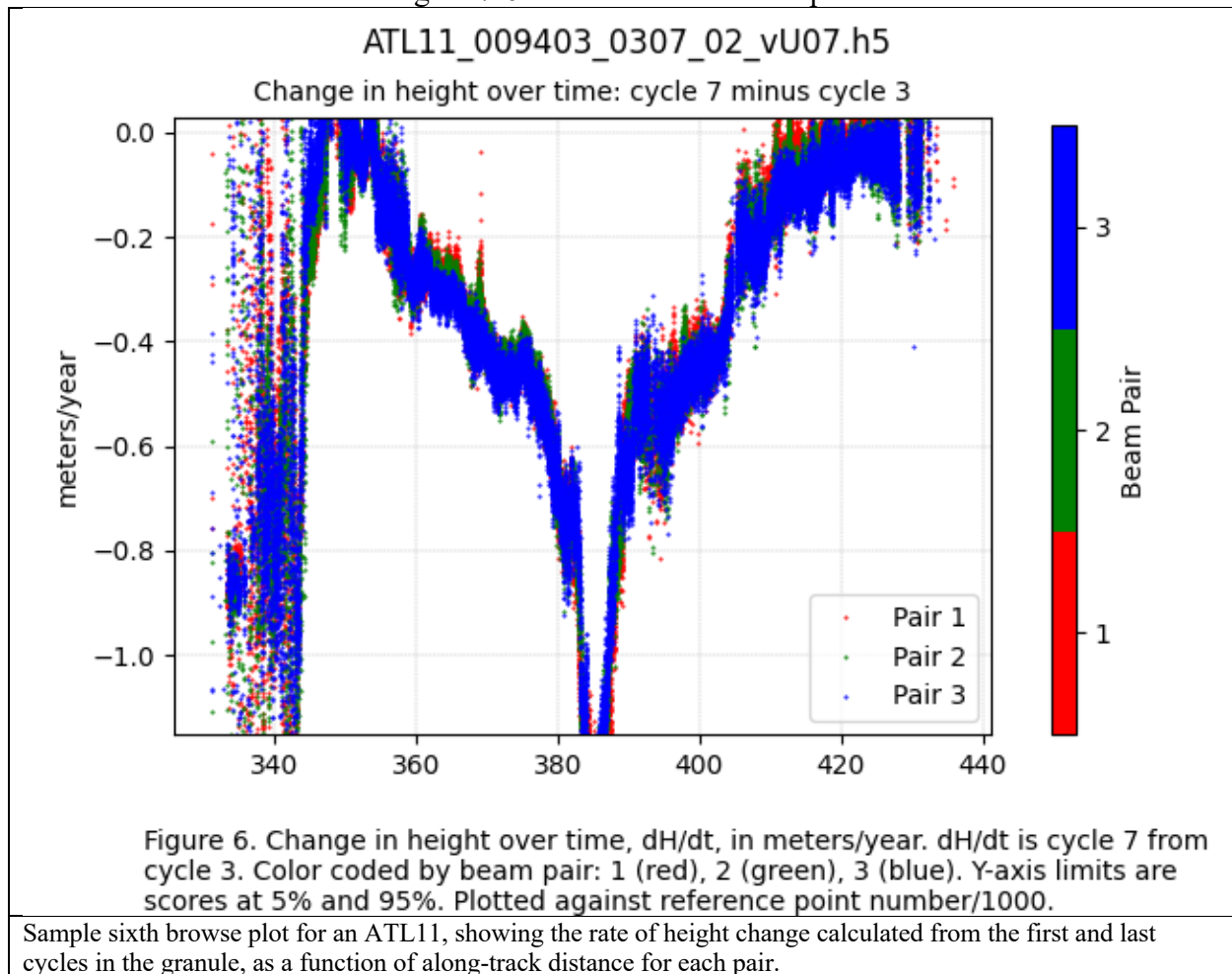
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Figure 7-6. Sixth ATL11 browse plot



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The changes in height with time, dH/dt , in meters/year are plotted in Figure 7-6. The calculation differences the first and last cycles with valid height data ($/ptx/h_corr$) divided by the associated time differences ($/ptx/delta_time$). The change in heights for pair 1 are in red, for pair 2 are in green and for pair 3 are in blue. The y-axis limits are set using the python `scipy.stats.scoreatpercentile` method with limits of 5% and 95%. The x-axis is reference point number ($/ptx/ref_pt$) divided by 1000 for a cleaner plot. Text of 'No Data' is printed in the panel if there is only one cycle with valid data, or if the first and last cycles with valid data have no common reference point numbers. This figure is called `dHdt` in the `BRW.h5` file.

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Glossary/Acronyms

ASAS	ATLAS Science Algorithm Software
ATBD	Algorithm Theoretical Basis Document
ATLAS	ATLAS Advance Topographic Laser Altimeter System
CDF	Cumulative Distribution Function
DEM	Digital Elevation Model
GSFC	Goddard Space Flight Center
GTs	Ground Tracks
ICESat-2	Ice, Cloud, and Land Elevation Satellite-2
IKR	I Know, Right?
MABEL	Multiple altimeter Beam Experimental Lidar
MIS	Management Information System
NASA	National Aeronautics and Space Administration
PE	Photon Event
POD	Precision Orbit Determination
PPD	Precision Pointing Determination
PRD	Precise Range Determination
PSO	ICESat-2 Project Science Office
PTs	Pair Tracks
RDE	Robust Dispersion Estimate
RGT	Reference Ground Track
RMS	Root Mean Square
RPTs	Reference Pair Tracks
RT	Real Time
SCoRe	Signature Controlled Request
SIPS	ICESat-2 Science Investigator-led Processing System
TLDR	Too Long, Didn't Read
TBD	To Be Determined

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