ICESat-2 Data Comparison User's Guide Release 6

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ICESat-2 Data Comparison User's Guide for Rel006

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Abstract

This document describes methods for making comparisons of release 006 ICESat-2 data products with results and data from other satellite missions. Included are considerations regarding reference systems, geophysical correction modeling, and underlying assumptions. Transformations are outlined to assist the user community to effectively apply ICESat-2 data in comparisons with independent sources.

Preface

This document provides a guide specifically aimed to assist rel006 ICESat-2 data users to properly transform ICESat-2 data products into reference systems adopted by other satellite missions, altimetric or otherwise. Initial chapters provide step-by-step transformational processes involving reference systems and considerations for geophysical corrections. Later chapters provide details on the definitions of various mission reference systems, and discuss underlying assumptions behind modeling choices. A Glossary is provided to aid in gaining an appreciation of geodetic and geophysical correction modeling terminology, used throughout.

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1. Introduction

The ICESat-2 user community possesses very diverse and unique scientific backgrounds, including glaciology, oceanography, atmospheric sciences, hydrology, biogeography and geodesy. There are users with expertise across a whole host of scientific sub-disciplines desiring to compare ICESat-2 data with results and data products from other measurement systems, spaceborne or otherwise. Additionally, there are users interested in quantifying cross-mission instrument and measurement biases.

Appreciating that some of the user community has less exposure to the geodetic underpinnings of the ICESat-2 mission or satellite laser altimetry, this document attempts to provide suggestions and guidance with various transformations and data treatments to assist users in making appropriate comparisons of ICESat-2 results with other satellite mission measurement systems. Other height measurement missions considered include Cryosat-2, and the original ICESat mission, herein called ICESat(GLAS). Other missions can be added to later versions of this document, by request of the user community.

Chapter 2 provides considerations and discussions with regard to height systems, and their relation to the treatment of the permanent tide, critical for inter-comparing mission specific data sources. Chapter 3 presents primary reference system (ellipsoid) transformations, providing alternatives for making coordinate conversions and ellipsoid transformations. Chapter 4 provides tables of reference system parameters for each mission. Chapter 5 provides geophysical correction definitions and parameters adopted for each mission. A Glossary is provided as an appendix, aiding in formal definitions of geodetic and geophysical terminology used throughout. Illustrations are provided to aid in conceptual understanding.

2. Height Comparisons and Tidal Systems

In this section, some basic concepts are presented in order to facilitate comparisons of ICESat-2 heights with those from other sources or space missions. Clear definitions and distinctions between various tidal systems and their respective treatment of the permanent tide is of utmost importance.

2.1 Tidal Systems

In this context, the term "tidal system," refers to how the permanent tide is treated and/or applied to the geoid model and/or for the solid earth tide geophysical correction. Essentially, there are three tidal systems. The discussion, herein, will mainly address two: The *mean tide system* and the *tide free system*. A third tidal system is known as the *zero tide system*. For further discussion on tidal systems, see, e.g., *Mäkenin & Idhe* (2009).

The presence of the Sun and Moon cause a tidal attraction that has periodic and non-periodic components deforming the shape of the geoid (Earth's geopotential) as well as it's crust. The non-periodic, zero frequency, or time-averaged, deformation is termed the *permanent tide*. The presence of the non-periodic part of the solar and lunar gravitational fields causes the planet's shape to bulge slightly more (increasing the flattening) than the case if the Sun and Moon were "removed" (i.e., set to an infinite distance). An exaggerated profile view of this increased bulge is shown in Figure 2-1. Thus, the permanent tide causes the Earth's shape to be slightly compressed at the poles, and extended in the equatorial plane.



Figure 2-1. Concept of mean tide and tide free systems; profile along a meridian line. Exaggerated bulge is not drawn to scale.

The amount of deformation the geoid and crust endures, due to the permanent tide, is a function of latitude (φ), and two scaling parameters (k_2 and h_2), respectively, known as *Love numbers*.

The **mean tide system** essentially involves anything actually measured in the real world, where the effect of the permanent tide is present. Thus, it is natural to measure, for example, the instantaneous sea level, relative to a reference surface (e.g., the geoid) that implicitly <u>includes</u> <u>deformation caused by the permanent tide</u>.

The **tide free system** (also known as the non-tidal system) is such that <u>the permanent tide</u> <u>deformation is eliminated</u> from the shape of the Earth, both in terms of a direct (gravitational) and indirect (surface or crustal) response.

2.2 Geoid Considerations for Release 006

Starting with release 004, the geoid height values are provided in the *tide free system*. These are transformed *into* the mean tide system by the equation:

geoid_free2mean =
$$0.1287 - 0.3848 \sin^2 \varphi$$
 (meters)

Where the degree-2 Love number, $k_2=0.3$ is implicit in the equation, and j is the latitude.

The geoid_free2mean term is critically important in rel006 ATL03 and upper-level processing. This term is provided at the same 20m posting as the geoid values. This free2mean term is *added* to the tide free system geoid height to form a mean tide system geoid height. Combining these steps, for every photon in a 20m along-track interval, the orthometric height, relative to the mean tide system is now given as:

Where geoid is now in the *tide free system*. The general equation is given by:

$$H_{ortho} = H_{ellipsoid} - H_{mean \ tide}^{geoid} = H_{ellipsoid} - (H_{tide \ free}^{geoid} + geoid_free2mean)$$

Figure 2-2 illustrates the numerical behavior of the geoid free2mean term, to transform a tide free system geoid height into the mean tide system. It can be seen from the figure that, at the equator, orthometric heights, computed on the basis of the mean tide geoid, will be lower than their corollary, computed on the basis of the tide free geoid. Conversely, at the poles, orthometric heights, computed on the basis of the mean tide geoid, will be higher than their corollary, computed on the basis of the mean tide geoid, will be higher than their corollary, computed on the basis of the mean tide geoid. The permanent tide effect is zero at a latitude of $\pm 35.2644^{\circ}$. In most cases, when re-referencing photon heights from the ellipsoid to the geoid is required, a mean tide system geoid should be used.





2.3 Solid Earth Tide Considerations

Similar to the geoid, the solid earth tides can be provided in a tide free system or in a mean tide system. On ATL03, from rel001 onwards, the solid earth tides have been computed in a *tide free system*. The ATL03 algorithm directs that the solid earth tides (among other geophysical corrections, c.f., *Neumann et al.*, 2019) be applied to the initial (i.e., uncorrected, or raw) photon heights. Thus, the photon heights, h_ph, found on the ATL03 product (and used as input by upper-level products), *have been corrected* for the effect of the tide free system, solid earth tides.

This situation is especially useful for direct comparisons with ground-based, GNSS and GPS measurements, where heights are typically provided in a tide free system.

However, there may be occasions when users will prefer to work with heights relative to the ellipsoid, given in terms of the mean tide system (e.g., in comparisons with tide gauge, or other satellite altimetry data). In the case when the photon height, relative to the WGS-84 ellipsoid (*not* to the EGM2008 geoid), is required to be in the mean tide system, the following equation is applicable:

tide_earth_free2mean = $0.06029 - 0.180873 \sin^2 \varphi$ (meters)

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Where the degree-2 Love number, $h_2=0.609$ is implicit in the equation and φ is the latitude. The above equation is applicable when it is desired to transform earth tide values from tide free to mean tide systems. For rel006, values based on this equation are provided on ATL03 (and upper-level products). For rel001 through rel003, the user must compute this term and apply it, when appropriate. The ATL03 parameter is called tide_earth_free2mean, also located in the gtx/geophys_corr group, at the 20m posting. When needed and appropriate, this term is added to the tide free system solid earth tide value, in order to form a mean tide system solid earth tide.

Figure 2-3 shows the numerical behavior of the solid earth tide free2mean term, added, to transform a tide free system solid earth tide height into the mean tide system. From the figure, the following can be inferred: the solid earth tide at the equator in the mean tide system, will be greater than the tide free value; conversely, the solid earth tide at the poles in the mean tide system, will be less than the tide free system value. Again, at latitudes $\pm 35.2644^{\circ}$, the permanent tide effect is zero.





Application: To transform an ATL03 photon ellipsoid height (h_ph) to refer to the mean tide system solid earth tide, this equation is used:

 $h_{ellipsoid_mean_sys_SE_tide} = h_ph - tide_earth_free2mean$

In this case, a subtraction is made since ATL03's h_ph parameter, by default, has the tide free system solid earth tide applied. In other words, given that h_ph = h_ph_{raw} - tide_earth_free_sys, where h_ph_{raw} is the photon height relative to the ellipsoid without any solid earth tide applied, and where tide_earth_free_sys is defined as being in the tide free system, then, with tide_earth_mean_sys = tide_earth_free_sys + tide_earth_free2mean, we get

hellipsoid_mean_sys_SE_tide = h_ph_raw - (tide_earth_free_sys + tide_earth_free2mean)
= (h_ph_raw - tide_earth_free_sys) - tide_earth_free2mean)
= h ph - tide free free2mean.

3. Geodetic Transformations

This chapter provides instructions on how to transform ICESat-2 height products from their adopted reference system into reference systems adopted by other missions. Definitions of fundamental reference system parameters are given in Chapter 4. The glossary provides assistance in clearly defining the technical terminology adopted here, shown in italics.

The main consideration of this chapter is to describe methods required to transform heights and latitudes, that are given relative to various surfaces. The foundational surface is that of an *ellipsoid* of revolution that mathematically approximates the shape of the Earth. An important secondary surface is mean sea level, approximated by the *geoid*, which, itself, is an *equipotential surface*.

These two primary height reference surfaces are typically specified by each altimetric mission, and are tabulated in Chapter 4.

3.1 Transformations Between Reference Ellipsoids

All altimetric missions utilize ellipsoids centered at the center of mass of the Earth, and oriented along the same axes (geocentric). The problem can be stated as such: Given geodetic coordinates in one ellipsoidal system (A), it is desired to transform them into another ellipsoidal system (B).

$$(\varphi, \lambda, h)_A \xrightarrow{T} (\varphi, \lambda, h)_B$$

Translations and rotations between ellipsoids and datums are not considered here. Specific realizations of the International Terrestrial Reference Frame (ITRF) may need consideration, by the user (cf., section 3.2).

There are several ways to make transformations between geocentric ellipsoidal reference systems. Two methods are described, here. The first way is to apply a **two-step conversion**, where one first converts geodetic coordinates (φ, λ, h) into Cartesian coordinates (X, Y, Z) using the parameters from the first ellipsoid (semi-major axis, *a*, and the inverse flattening, 1/f); then make an inverse conversion, from Cartesian (X, Y, Z) back into geodetic coordinates (φ, λ, h) , using the second ellipsoid's parameters. A second way to make the transformation is to use a **differential projective transformation procedure**.

Additionally, there are software procedures freely available that facilitate making these kinds of coordinate conversions and transformations. For example, the Generic Mapping Tools (GMT; https://www.generic-mapping-tools.org/) is able to make ellipsoid transformations within the command, mapproject. Within the Matlab environment, a very useful package, called Geodetic Transformations (by Peter Wasmeier) is available from Matlab's file exchange. Within the python environment, pyproj provides a useful interface to PROJ transformations.

3.1.1 Two-Step Conversion

This process involves two steps of coordinate conversion, involving the computation of, and utilization of an intermediate set of three-dimensional, Cartesian coordinates:

$$(\varphi, \lambda, h)_A \xrightarrow{T} (X, Y, Z) \xrightarrow{T} (\varphi, \lambda, h)_B$$

This begins by computing the *radius of curvature* in the prime vertical, N, (i.e., perpendicular to the local meridian) for the specific location, using the ellipsoid parameters $(a, f)_A$ for which the geodetic coordinates (φ, λ, h) exist (for ellipsoid A),

$$N = \frac{a}{(1 - e^2 sin^2 \varphi)^{1/2}}$$

Where, e^2 is the square of the first eccentricity, $e^2 = (a^2 - b^2)/a^2 = 1 - (1 - f)^2$. Then, the 3D-space Cartesian coordinates can be computed via (click here for an on-line tool):

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} (N+h)cos\varphi cos\lambda \\ (N+h)cos\varphi sin\lambda \\ ((1-e^2)N+h)sin\varphi \end{bmatrix}$$

Once the Cartesian coordinates are computed, then, using the parameters for the ellipsoid *B*, $(a, f)_B$, the inverse transformation is applied. This is trivial for the longitude, as $tan\lambda = Y/X$, which, since the origin and orientation are common between the two ellipsoids, the longitude for the second ellipsoid will be the same as the longitude for the first.

Computing the latitude (ϕ) and height (h) is often done in an iterative way, but can also be done in a nearly direct way. One such method is outlined here (Vermeille's method). An on-line Cartesian to Geodetic tool can be accessed <u>here</u> (not all ellipsoids are represented).

The method computes a set of intermediate parameters, using parameters that pertain to the ellipsoid *B*;

$$p = \frac{X^2 + Y^2}{a^2}$$
$$q = \frac{1 - e^2}{a^2} Z^2$$
$$r = \frac{p + q - e^4}{6}$$
$$s = e^4 \frac{pq}{4r^3}$$
$$t = \sqrt[3]{1 + s} + \sqrt{s(2 + s)}$$
$$u = r\left(1 + t + \frac{1}{t}\right)$$
$$v = \sqrt{u^2 + e^4q}$$

$$w = e^{2} \frac{u + v - q}{2v}$$
$$k = \sqrt{u + v + w^{2}} - w$$
$$D = \frac{k\sqrt{X^{2} + Y^{2}}}{k + e^{2}}$$

Then, geodetic latitude and height above the ellipsoid is given by,

$$\varphi = 2 \arctan \frac{Z}{D + \sqrt{D^2 + Z^2}}$$
$$h = \frac{k + e^2 - 1}{k} \sqrt{D^2 + Z^2}$$

3.1.2 Differential Projective Transformation

The differential projective transformation procedure computes changes to latitude and height as a function of the change in semi-major axis ($da = a_B - a_A$) and flattening ($df = f_B - f_A$), utilizing the ellipsoid parameters from ellipsoid A. The resulting change in latitude ($d\phi$) and height (dh) are applied to the coordinates ($\phi_i h)_A$ to yield ($\phi_i h)_B$.

$$(\varphi, h)_A \xrightarrow{T[f(da, df)]} (d\varphi, dh); \varphi_B = \varphi_A + d\varphi \text{ and } h_B = h_A + dh_A$$

$$d\varphi = \frac{1}{M+h} \left[\frac{e^2 \sin\varphi \cos\varphi}{W} da + \sin\varphi \cos\varphi (2N + e'^2 M \sin^2 \varphi) (1-f) df \right]$$
$$dh = -W da + \frac{a(1-f)}{W} \sin^2 \varphi df$$

Where $W = \sqrt{1 - e^2 \sin^2 \varphi}$, $M = a(1 - e^2)/W^3$ (the radius of curvature in the meridian), N = a/W (the radius of curvature in the prime vertical, perpendicular to the meridian plane), and the square of the second eccentricity, e'^2 , is given by $e'^2 = (a^2 - b^2)/b^2$.

3.1.3 Example of Ellipsoid Transformation

As a numerical example, given geodetic coordinates in the WGS-84 ellipsoid system (such as ICESat-2 geolocated photons), and we wish to transform these into the Topex/Poseidon (T/P), ellipsoid system (such as ERS-1 geolocated radar waveforms). [ITRF considerations are described in Sect. 3.2.2.]

Given these example geodetic coordinates, in WGS-84:

$$\begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix}_{WGS-84} = \begin{bmatrix} 47^{\circ} \\ 15^{\circ} \\ 1200m \end{bmatrix}$$

Using the **two step method**: first convert the WGS-84 geodetic coordinates into Earthcentered Cartesian coordinates (units: meters):

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 4209993.6131 \\ 1128064.3888 \\ 4642642.4133 \end{bmatrix}$$

Next, applying the inverse conversion, utilizing the T/P ellipsoid parameters, we compute new latitude, longitude and height to be:

$[\varphi_1]$		[47.000000123°]
λ	=	15.000000000°
$\lfloor h \rfloor_{T/I}$	Р	1200.7073 <i>m</i>

The **differential projective transformation** starts with WGS-84 coordinates, above, utilizing the ellipsoidal difference quantities, da = -0.7m and df = 0.0000000251315, we compute

thus yielding the same result, to within a tenth of a millimeter, as the two step method, when $d\varphi$ and *dh* are added to the original WGS-84 latitude and height.

3.2 Mission Specific Ellipsoid Transformations

This section provides information needed for transforming coordinates adopted by one mission into coordinates adopted by the ICESat-2 mission. In these cases, adoption of International Terrestrial Reference Frame (ITRF) definitions becomes important when making transformations. The ITRF transformations from ITRF2014 to earlier realizations of the ITRF are given in Appendix 2.

3.2.1 Between ICESat-2 and Cryosat-2

Since both Cryosat-2 and ICESat-2 both adopted the WGS-84 ellipsoid, and are placed in the same realization of the ITRF (ITRF2014), no transformation for ellipsoid is needed. Positions are in the same ellipsoidal reference system between missions.

3.2.2 Between ICESat(GLAS) and ICESat-2

The reference systems adopted by each mission differ in two main ways. First, their respective ellipsoid parameters differ; and second, the coordinate frame for each mission are given across two different realizations of the ITRF. ITRF-2008 was the final adopted terrestrial reference frame for ICESat(GLAS) [*Schutz & Urban*, 2014, Appendix B]. The latest release (rel 34) of ICESat(GLAS) data is given in the ITRF2008 realization.

Ellipsoid Parameters. ICESat(GLAS) adopted the Topex/Poseidon (T/P) ellipsoid (cf., Chapter 4 for parameters) which is an ellipsoid that can fit wholly inside the WGS-84 ellipsoid (Figure 3-1). In other words, the T/P ellipsoid is ~70cm smaller than WGS-84. Generally, without consideration of ITRF translations and rotations, heights above the T/P ellipsoid will be

70 to 71.37cm larger than WGS-84 ellipsoidal heights for the same point (P) in threedimensional space (as seen in the example, below).

ITRF Considerations. Given that the ICESat(GLAS) data is geolocated within ITRF2008, then coordinates require an additional translation to bring them into ICESat-2's ITRF2014 framework. Appendix 2 provides the necessary transformation parameters. Appendix 2's eq. 1 must be reversed to transform ICESat(GLAS) ITRF2008 (XS, YS, ZS) coordinates into the ICESat-2 ITRF2014 frame (X, Y, Z).

[X]	[<i>XS</i>]	$\begin{bmatrix} T_x \end{bmatrix}$	[D	$-R_z$	R_y	[<i>XS</i>]
Y =	= YS -	$ T_y =$	R_z	D	$-R_x$	YS
$\lfloor_Z \rfloor$	$\lfloor_{ZS} \rfloor$	$\left[T_{z}\right]$	$\left[-R_{y}\right]$	R_x	D	$\lfloor ZS \rfloor$

Use of this equation is made with Cartesian form of the coordinates. Coordinate transformations needed to convert from geodetic to Cartesian, and back again, were described in Sect. 3.3.1.

In this specific case, no rotations are involved, and the above equation, reduces to

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} XS \\ YS \\ ZS \end{bmatrix} - \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} - D \begin{bmatrix} XS \\ YS \\ ZS \end{bmatrix}$$

3.2.2.1 Example

In this example, let's say we are given a ICESat(GLAS) geolocated observation, that refers to the Topex/Poseidon ellipsoid, in the ITRF2008 frame with an observational epoch of 2005.3 of:

$$\begin{bmatrix} \varphi \\ \lambda \\ h \end{bmatrix}_{T/P, ITRF2008, 2005.3} = \begin{bmatrix} 42^{\circ} \\ 10^{\circ} \\ 210m \end{bmatrix}$$

The first step is to convert these coordinates into Earth-centered fixed Cartesian form using Section 3.1.1, yielding (in meters):

$$\begin{bmatrix} XS \\ YS \\ ZS \end{bmatrix} = \begin{bmatrix} 4675034.5692 \\ 824334.7303 \\ 4245743.8709 \end{bmatrix}$$

Next, the values of the scale, D, and translations, (T_x, T_y, T_z) , are adjusted to place them into the 2005.3 epoch (values obtained from Appendix 2);

$$D_{2005.3} = D_{2010} + \dot{D}(2005.3 - 2010) = -0.02 + 0.03(-4.7) = -0.161 \, ppb$$

$$T_{n,2005.3} = T_{n,2010} + \dot{T}_n(2005.3 - 2010) = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}_{2005.3} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}_{2010} + (2005.3 - 2010) \begin{bmatrix} \dot{T}_x \\ \dot{T}_y \\ \dot{T}_z \end{bmatrix}_{2005.3} = \begin{bmatrix} 1.6 \\ 1.9 \\ 2.4 \end{bmatrix} + (-4.7) \begin{bmatrix} 0 \\ 0 \\ -0.1 \end{bmatrix} = \begin{bmatrix} 1.6 \\ 1.9 \\ 2.87 \end{bmatrix} \text{mm}$$



Figure 3-1. Difference between ICESat(GLAS), Topex/Poseidon (T/P, in blue) and ICESat-2, WGS-84 (in red) ellipsoids.

Applying the transformation, from ITRF2000, epoch 2005.3 to ITRF2014, epoch 2010 yields:

[4675034.5692] 0.0016 4675034.5692 [4675034.5692] [0.002353] 824334.7303 824334.7303 0.0019 $-(-0.161 \times 10^{-9})$ 824334.7303 0.002033 = = 4245743.8709 0.00287 [4245743.8709] L4245743.8709 L0.003554 4675034.5668 824334.7283 4245743.8674

Converting Earth-centered fixed Cartesian coordinates back into WGS-84 geodetic coordinates, in the ITRF2014 (2010 epoch) reference system, yields:

۲ ^φ ۱	[41°59′59.999"]
$ \lambda =$	9°59′59.999"
$[h]_{WGS-84,ITRF2014,2010}$	[209.2895 <i>m</i>]

Examining the two sets of coordinates (transformed and un-transformed), it can be seen that the latitude and longitude changed less than one milli-arcsecond, but the transformed height is now 0.7105m lower than original un-transformed value. This is expected, given that the surface of the WGS-84 ellipsoid is between 70 and 71.37 cm above that of the Topex/Poseidon ellipsoid (Figure 3-1). In this particular example, the ITRF transformation (to epoch 2010) only amounted to 2mm to 3.6mm.

4 Reference Systems & Geoids

The tables in this chapter provide the fundamental values associated with the reference system adopted by various satellite missions.

Table 4.1 provides the mission ellipsoid specification. The ellipsoid is a mathematical definition, as a surface of revolution closely fitting the actual shape of the Earth. The semi-major axis is an approximate radius of the Earth in the equatorial plane. The flattening describes the degree of oblateness.

Mission	Ellipsoid	Semi-major axis (a)	Inverse flattening (1/f)
ICESat-2	WGS-84 (G1150)	6378137.0 m	298.257223563
CryoSat-2	WGS-84	6378137.0 m	298.257223563
ICESat(GLAS)	Topex/Poseidon	6378136.300 m	298.257

Table 4.1. Ellipsoid specifications by mission.

Other auxiliary parameters can be computed from these values. For example, the semi-minor (polar) axis for WGS-84 is given by,

$$b = a(1 - f) = 6378137 \left(1 - \frac{1}{298.257223563}\right) = 6356752.3142 \, m.$$

And the square of the first eccentricity is given by,

$$e^2 = 1 - (1 - f)^2 = 0.00669437999.$$

Table 4.2 provides geoid specification by mission. The geoid is a level surface, or equipotential surface that represents the actual shape of the planet, neglecting topography/bathymetry.

Table 4.2. Geoid specification by mission.

Mission	Geoid	Tidal System	Range of Values
ICESat-2	EGM2008	Mean tide	-106.7843 to 85.9602 m
CryoSat-2	EGM96	N.A.	N.A.
ICESat(GLAS) beginning with rel 31	EGM2008	Mean tide	-106.7843 to 85.9602 m

The mean tide system means that the geoid heights include the effect of the geoid deformation due to the permanent (time-invariant) tide, caused by the gravitational pull from the Moon and Sun. The tide free system does not include the permanent tide. See Section 2.3 for

special considerations regarding the utilization of ICESat-2's geoid as a reference surface (e.g., to approximate mean sea level).

As mentioned above, the ellipsoid is an approximate mathematical description of the shape of the planet. The geoid provides an even better representation of the sea level surface. The global variations in the EGM2008 geoid are shown in Figure 4-1. These variations are due to variations in the distribution of planetary mass at depth. It is worth noting that on ATL03, photon heights are given relative to the WGS-84 ellipsoid. This means that photon heights for locations at or near sea level can be negative (less than zero).



Figure 4-1. EGM2008 geoid heights, relative to the WGS-84 ellipsoid. Notable are the Indian low, the East Indies high and the North Atlantic high.

5 Considerations For Geophysical Corrections

Satellite altimetry operates on the principle of measuring a round-trip travel time traversed by a signal (laser or radar) emitted from a satellite at altitude, reflected by a real-world surface, and detected by receiver systems on-board the satellite. The determination of a height, at the reflection point on the planetary surface, must be understood as a *range* from the observation platform (satellite or airborne) to an instantaneous, typically dynamic, surface. For ICESat-2, reflective surfaces can include oceans, inland water bodies, solid ground, land and sea ice, vegetation and man-made structures.

Figure 5-1 provides a conceptual schematic to illustrate the various reference systems (covered in Chapters 2 through 4) and dynamic processes that take place at the time when a

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range measurement is observed. These processes are grouped under the heading, *geophysical corrections*. For example, in order to yield an estimate of the mean sea surface, a whole host of well-modeled, time-varying effects must be accounted for. The present section describes these effects and how they are represented, and/or applied on the ATL03 product. Upper level products may apply some of these in a selective fashion. The reader is encouraged to consult respective higher-level ICESat-2 ATBDs for further information, as it's not possible to cover every possible scenario.

Generally, an instantaneous surface height, H_P (or, in ATL03 terms, a photon event height, cf., *Neumann et al.*, 2019, section 6.1), is defined as follows:

$$H_P = H_{ellipsoid}^{sat} - H_{surface}^{sat}$$

Where $H_{ellipsoid}^{sat}$ is the distance to the satellite above the ellipsoid (determined by precision orbit determination); and $H_{surface}^{sat}$ is the re-tracked range (i.e., the actual measurement, corrected for atmospheric range effects). Next, the photon event height, H_P , is corrected for an *i*-number of temporally and spatially varying geophysical effects (tides, loading, etc., as listed below), that are represented by C_i , which are, by convention, consistently subtracted via:

$$H_{gc} = H_P - \sum C_i$$

where H_{gc} are geophysically-corrected photon event heights, referring to the WGS-84 ellipsoid. These geophysically-corrected photon event heights are the gtx/heights/h_ph values on the ATL03 data product.

The geophysical corrections necessary over each surface-type are given in Table 5-1.

•	Photon-Level Product (ATL03)
	Ocean loading (section 5.2)
	Solid Earth tide (section 5.3)
	Solid Earth & ocean pole tide (section 5.4)
•	Land Ice, Land, and Inland Water (ATL06, ATL08 & ATL13):
	No additional corrections
•	Sea Ice (ATL07/10):
	Referenced to Mean Sea Surface (see Kwok and Morison [2016])
	Ocean tide (section 5.1)
	Long period equilibrium ocean tide (section 5.1)
	Inverted barometer (IB, section 5.5)
•	Ocean (ATL12):
	Ocean tide (section 5.1)
	Long period equilibrium ocean tide (section 5.1)
	Dynamic Atmospheric Correction (IB + wind effects, section 5.5)

Table 5-1. Geophysical corrections as applied by surface type.



odified from: Tapley, B. D. & M-C. Kim, Applications to Geodesy, Chapt. 10 in Satellite Altimetry and Earth Sciences, ed. by L-L. Fu & A. Cazenave, Academic Press, pp. 371-406, 2001.

Figure 5-1. Schematic of geophysical corrections utilized in satellite altimetry.

Since some of the corrections are applicable across all surface types, they are applied within ATL03, at the photon height level. These include: ocean loading; solid earth pole tides; ocean pole tides; and solid earth tides. The total column atmospheric range-delay term is also applied within ATL03, but is not discussed, here.

5.1 Ocean Tide Correction, Including Long-Period Equilibrium Ocean Tide

The ocean tides and long-period equilibrium tides are not applied to the photon heights on *ATL03* and are provided only as reference values at the geolocation segment rate. Higher level products may (or may not) have applied these values.

Ocean tides account for about 70% of the total variability of the ocean surface at daily and half-daily periods (diurnal and semi-diurnal). The effects of tides vary regionally; open ocean areas typically have smaller amplitudes ($\pm 0.3 \text{ m r.m.s}$) than continental shelves and coastal regions (which can increase to several meters, or more).

Ocean tide models quantitatively describe the time-variant changes of sea level due to gravitational attraction by the Sun and the Moon. The models are applicable for any point in the oceans, as well as on sea ice and on ice shelves.

Ocean tide values are computed using the GOT4.8 ocean tide model, with long-period equilibrium tides being computed independently via the LPEQMT.F Fortran routine where fifteen tidal spectral lines from the Cartwright-Tayler-Edden tables are summed. GOT4.8 was selected since ICESat-2's Precision Orbit Determination effort relies on the GOT4.8 model (cf. ICESat-2 POD ATBD).

The computations for ocean tide reference values, as well as long period equilibrium tides are performed by computing a modeled tidal height at each reference photon (~20 meters, along-track) latitude and longitude, along with its corresponding time, *t*. For locations where the ocean tide is undefined (e.g., over land), the value of the ocean tide correction is set to an invalid value (approximately 3.4E38). Both parameters are found on ATL03 in gtx/geophys_corr/tide_ocean and gtx/geophys_corr/tide_equilibrium.

5.1.1 Resolution and Edge Performance Limits of the GOT4.8 Ocean Tide Model

There are some nuances in the GOT4.8 ocean tide model that some ICESat-2 data users should be aware of. The model was developed to be of greatest utility in the open ocean, thus performance of the model diminishes in near-coastal and shallow-sea regions. The resolution of the model is ½° which causes very localized variations to remain largely unmodeled. This subsection aims to provide illustrations of the extent and level of this diminished performance.

A root sum square standard deviation difference analysis of seven data-constrained ocean tide models (*Stammer et al.*, 2014) indicates regions with ocean depths < 1000m have lower consistency (higher variance) across the models than regions with ocean depths > 1000m, at a level of being six times worse. Figure 1 of *Stammer et al.* (2014), shows that the highest variance between all seven models (M_1 and K_1 components) takes place in regions including the Artic

(around the 75° latitude), a latitude band surrounding Antarctica (-75°), along the northern coast of Australia, through the East Indies, and along the coasts of East Asia.

Detailed Examples of Tide Model Edges. A model limitation to the GOT (perth3.f) modeling software is that the underlying harmonic grids are provided at a $\frac{1}{2}^{\circ}$ resolution with a built-in mask that prohibits/allows estimation in a very generalized manner. Figure 5-2 shows examples of ocean tide evaluations for 5°x5° regions, computed at 1 arc-minute resolution, displaying how the software treats coastal edges in selected parts of the world.

Figure 5-2a shows a region off the NW coast of Australia (near Darwin), possessing a fairly large tidal gradient surrounding the Tiwi islands running from -157 cm (NW of Melville Island) and growing to 153 cm in the Van Diemen Gulf. Gray zones are where the software masks any estimate of the ocean tide. The edge often enters the actual near-shore ocean, and occasionally includes land zones, such as that seen in the lower left corner (western shore of the Joseph Bonaparte Gulf).

Figure 5-2b shows a region near Shanghai, China. Low tides are seen along the Yellow Sea coast, and high tides south of Hangzhou Bay. Many islands and peninsulas along this coast have had ocean tides computed by the model.

Figure 5-2c shows a relatively calm tidal region in northeast Hudson Bay, Canada. The generalized edge of the model excludes some regions of the bay itself, and Mansel Island has, for the most part, ocean tides computed well inland.

Figure 5-2d shows tides in the Irish Sea varying from -4.46 to 3.6 m along the south and north coasts of Wales, respectively. The entire Isle of Man, along with islands and peninsulas on the Scottish coast, have values of the ocean tides estimated by the model. Yet the Solway Firth and, to the south, Morecambe Bay and the shallow seas off-shore from Blackpool & Southport, UK lack tidal modeling.

Maps found in Figure 5-2 clearly show potential difficulties in near-coast and shallow-sea tidal modeling based on GOT4.8. Users must be aware of and proceed with caution when making studies in such regions.

Arctic and Antarctic Edges. The resolution of the GOT4.8 tidal field in the Arctic and Antarctic edges are shown in Figure 5-3. For the Arctic, the grid resolution varies across two latitude bands. Between latitudes 50° and 70° the resolution is 10 arc-minutes in both lat/lon. Above 70° , the spacing is 15 arcmins (e-w) and 10 arcmins (n-s). For the Antarctic, the grid resolution varies across two latitude bands. Between latitudes -50° and -70° the resolution is 10 arc-minutes in both lat/lon. Above 70° , the spacing is 15 arcmins (e-w) and 10 arcmins (n-s).

Unfortunately, at this resolution, it is not particularly easy to spot edge issues, but under close examination, the GOT4.8 tidal model occasionally excludes bays and inlets, and often includes islands and peninsulas.



Figure 5-2. Four 5°x5° ocean tidal field examples computed by GOT4.8 software for June 4, 2019 at 1200h. Upper left (a) off the coast of NW Australia; upper right (b) China coast, near Shanghai; lower left (c) Northeast coast of Hudson Bay, Canada; and lower right (d) the Irish Sea. Color scales are relative to each region. Computation resolution is 1 arc minute, with the mask being shown as gray dots.



Figure 5-3. Polar stereographic maps showing where the GOT4.8 estimates tidal values (green dots). Top: Arctic; Bottom: Antarctic. White areas are where the GOT4.8 model is internally masked within the software.

5.1.2 Ocean Tides for CryoSat2

The tide model used for ocean tide corrections is the FES2004 model (parameter: ocean_tide_01), which has latitude coverage from -90 to +90 degrees and a latitude/longitude resolution of 0.125 degrees (*ESA*, 2014 & *Lyard et al.*, 2006). The Ocean Tide does not include the ocean loading tide or the long-period equilibrium tide, but are provided separately. The long-period equilibrium tide can be found in the ocean_tide_eq_01 parameter.

5.2 Ocean Loading Correction

This correction removes the deformation of the Earth's crust due to the weight of overlying ocean tides. As the tides rise and fall, mass is added or lost in the water column and this mass redistribution cause loading of the ocean bottom.

Ancillary ocean loading phase and amplitude files corresponding to the GOT4.8 ocean tide model provide the basis for computing corrections for ocean loading for ICESat-2. These files include ten major tidal constituents (q_1 , o_1 , p_1 , s_1 , k_1 , n_2 , m_2 , m_4 , s_2 and k_2) and sixteen minor tides ($2Q_1$, s_1 , r_1 , M_1 , c_1 , p_1 , f_1 , q_1 , J_1 , Oo_1 , $2N_2$, m_2 , n_2 , l_2 , L_2 , T_2).

For each reference photon (~ 20 meters, along-track), based on its location (latitude, longitude) and corresponding time, ocean load vertical displacements are computed. This value has been subtracted for all photon heights on ATL03. The parameter is found on ATL03 in gtx/geophys_corr/tide_load.

5.2.1 Ocean Loading for CryoSat2

The loading tide model used to compute these corrections is the FES2004 model, which has latitude coverage from -90 to +90 degrees and a step in latitude/longitude of 0.25 degrees (ESA, 2014).

5.3 Solid Earth Tide Correction

The correction for solid earth tides considers the deformation (elastic response) of the solid earth (including the sea floor) due to the attractions of the Sun and Moon. Ninety-five percent of the tidal energy comes from the second-degree (semi-diurnal) tides. The procedure adopted by the ICESat-2 mission for the calculation of displacements due to solid earth tides follows that recommended by the IERS Conventions 2010.

A two-step process was followed by ATL03 in computing the solid earth tides: (1) displacements utilizing frequency-independent nominal values of the Love and Shida numbers, for the second and third degree of the tidal potential, are computed; (2) further corrections computed in the frequency domain. The second step takes account of the frequency-dependent deviations of the Love and Shida numbers and also of variations arising from mantle anelasticity.

As described in section 2.3, the solid earth tides reported on ATL03 are provided in the tidefree system. The parameter is found on ATL03 in gtx/geophys_corr/tide_earth. The conversion term needed to transform into the tide-free system values into mean tide system values is provided in rel006 by the tide_earth_free2mean parameter.

5.3.1 Solid Earth Tides for CryoSat2

The tide model used to compute these corrections is the Cartwright model (*ESA*, 2014 & *Cartwright & Edden*, 1973). Although not explicitly stated in CryoSat2 documentation, the solid earth tides are given in the mean tide system.

5.4 Solid Earth and Ocean Pole Tide Correction

Generally, the pole tide is a response of both the solid earth and oceans due to the centrifugal potential caused by small perturbations of the Earth's rotational axis (i.e., daily polar motion the Chandler wobble, x_p , y_p). The displacement, in the radial direction, utilizes Love number values appropriate to the frequency of the pole tide (c.f., eq. 7.26 in IERS, 2010)

The solid earth pole tide parameter is found on ATL03 in gtx/geophys_corr/tide_pole. The ocean pole tide parameter is found on ATL03 in gtx/geophys_corr/tide_oc_pole.

5.4.1 Solid Earth and Ocean Pole Tides for CryoSat2

The CryoSat-2 mission terms this correction as the *Geocentric Polar Tide correction*. The correction is derived using dynamic Instantaneous Polar Location files, which comprise the historical pole positions and are provided as daily dynamic files sourced from CNES via SSALTO. ESA Documentation lacks a complete description whether this correction consists solely of the solid earth component of the correction, or a combination of solid earth and ocean components (*ESA*, 2014). It is thought that only the former is present on data records.

5.5 Inverted Barometer (IB) Correction and Dynamic Atmospheric Correction (DAC)

The Dynamic Atmospheric Correction (DAC) is not applied to the photon heights on ATL03 and is provided only as reference values at the geolocation segment rate.

These two corrections, IB and DAC are inter-related, conceptually. The IB correction describes the response of the ocean's surface level caused by variations in atmospheric pressure (i.e., high and low pressure systems). The DAC correction contains IB as well as water mass momentum forcing driven by the highly variable wind stress field. In the most general sense, DAC provides a much more dynamic surface response than IB.

ATL03 provides reference values from the MOG2D DAC (2 Dimensions Gravity Waves model, available from AVISO). This DAC is available in six-hour increments, at a 0.25° geographic resolution. The ATL03 algorithm computes values, along-track, every ~20 m, that have been simultaneously interpolated, both geographically and temporally. The parameter is found on ATL03 in gtx/geophys_corr/dac.

On the ATL07, sea ice upper-level product, the IB is provided, and has been applied, at the segment output rate. The IB on ATL07 was derived from sea level pressure data found on ATL09, atmospheric product, as specified in the ATL07 ATBD. The IB parameter is found on ATL07 in gtx/sea_ice_segments/geophysical/height_segment_ib. The DAC parameter is found on ATL07 in gtx/sea_ice_segments/geophysical/height_segment_dac.

Comparisons of ATL07 with ATL12 (upper-level ocean product) can introduce the situation whereby ATL07 has had the IB correction applied, and ATL12 has had the DAC applied. These two corrections must be reconciled before attempting to make such a comparison.

5.5.1 DAC and IB for CryoSat-2

The CryoSat-2 mission provides MOG2D DAC values, over the ocean, only where there is no sea-ice cover. The IB correction is provided in SAR mode data, over sea ice and when the surface type is "Open Ocean". The source for the IB correction is ECMWF, as well as static S1 and S2 tide grids of monthly means of global amplitude and phase (*ESA*, 2014).

References

- Cartwright, D. E. and A.C. Edden, Corrected tables of tidal harmonics, *Geophysical Journal of the Royal Astronomical Society*, 33, 253-264, 1973.
- ESA, Geophysical Correction Application in Level 2 CryoSat Data Products, IDEAS-VEG-IPF-MEM-1288 Version 2.0, 2014.
- Kwok, R., and J. Morison, Sea surface height and dynamic topography of the icecovered oceans from CryoSat-2: 2011–2014, *J. Geophys. Res. Oceans*, 121, doi:10.1002/2015JC011357, 2016.
- Lyard, F. et al., Modelling the Global Ocean Tides: Modern Insights from FES2004, *Ocean Dynamics* 56 : 394-415, doi :10.1007/s10236-006-0086, 2006.
- Mäkinen, J and J. Ihde, The Permanent Tide in Height Systems, in *Observing our Changing Earth*, ed. by M.G. Sideris, IAG 133, 81-87, 2009.
- Neumann, T. et al., ATLAS/ICESat-2 L2A Global Geolocated Photon Data, Version 3, Boulder, Colorado USA. NASA National Snow and Ice Data Center Distributed Active Archive Center. doi: 10.5067/ATLAS/ATL03.003, 2019.
- Petit, G. and B. Luzum, IERS Conventions (2010), IERS Tech. Note 36, 2010.
- Schutz, B. E. and T. J. Urban, The GLAS Algorithm Technical Basis Document for Laser Footprint Location (Geolocation) and Surface Profiles, NASA TM-2014-208641, Vol. 12, 2014.
- Stammer, D. & 26 others, Accuracy Assessment of Global Ocean Tide Models, *Rev. Geophys.* 52, 243-282, doi:10.1002/2014RG000450, 2014.

Appendix 1 – Glossary

Ellipsoid – a mathematical construct, flattened at the poles, suitable for unequivocally describing horizontal and vertical positions. Ellipsoids are typically defined by two parameters: the semi-major axis (*a*) and the amount of flattening, f = (a - b)/a, typically described by its inverse, 1/f. An ancillary quantity is the first eccentricity, $e \equiv OF_1/a$ (see diagram for quantities), and is often used as a squared quantity, $e^2 = (a^2 - b^2)/a^2$.



Equipotential Surface – a surface of constant gravity potential. The *geoid* is but one of many such surfaces.

Flattening – parameter (f) describing the extent of rotational ellipsoid axial difference, defined by f = (a - b)/b, where a is the semi-major (equatorial) axis dimension and b is the semi-minor (polar) axis dimension.

Geoid – a level, or equipotential surface of the Earth's gravity field corresponding to sea level (free of ocean currents and other disturbances; such as ocean tides, barometric and wind effects, etc.).

Geophysical Corrections – a grouping of temporal (time varying) processes, originating gravitationally or by other forcing sources, that are modeled to such accuracy that they can be used to correct a range measurement for a variety of dynamic effects.

Love Numbers – unitless ratios that provide a measure of rigidity of a planetary body and the susceptibility of its shape to change in response to a tidal potential. The Love number, h, is the ratio of the elastic radial displacement of a mass element of the actual Earth to the radial displacement of the corresponding hypothetical fluid Earth, or, to put it another way, the height of the body tide to the height of the equilibrium (static) marine tide. The Love number, k, is the ratio of the additional potential produced by the redistribution of mass to the deforming potential.

Mean Tide System – the permanent tide is included in defining the shape of the Earth. The shape therefore corresponds to the long-time average under lunar- and solar-tidal forcing.

Radius of Curvature – on an ellipsoid, two mutually perpendicular normal sections whose curvatures are minimum and maximum. These are termed the principle normal sections: meridian (*M*); and prime vertical (*N*). Generally, $N \ge M$; but at the poles ($\varphi = \pm 90^{\circ}$), $M_{90^{\circ}} = N_{90^{\circ}} = a^2/b$; and at the equator ($\varphi = 0^{\circ}$), $M_{0^{\circ}} = b^2/a$, $N_{0^{\circ}} = a$.

Range – generally, half of the observed travel time (Δt) a signal takes to traverse the distance between an on-board emitter (laser or radar), a reflection point, and an on-board receiver, scaled by the speed of light (*c*). Range = $c(\Delta t/2)$, neglecting path delays.

Tide Free System – the permanent tide is eliminated from the shape of the Earth. From the potential field quantities (gravity, geoid, etc.) both the tide-generating potential, and the deformation potential of the Earth (the indirect effect) are eliminated. This corresponds to physically removing the Sun and the Moon to infinity.

Zero Tide System – eliminates the tide-generating potential but retains its indirect effect, i.e., the potential of the permanent deformation of the Earth.

Appendix 2. ITRF2014 Transformation Parameters

Source: https://itrf.ign.fr/doc ITRF/Transfo-ITRF2014 ITRFs.txt

SOLUTION	Tx	Ту	Tz	D	Rx	Ry	Rz	EPOCH
UNITS	-> mm	mm	mm	ppb	.001"	.001"	.001"	
	•	•	•	•	•	•	•	
RATES	Tx	ТУ	Tz,	D	Rx	Ry	Rz	
UNITS	> mm/y	mm/y	mm/y	ррь/у	.001"/y	.001"/y	.001"/y	
ITRF2008	1.6	1.9	2.4	-0.02	0.00	0.00	0.00	2010.0
rates	0.0	0.0	-0.1	0.03	0.00	0.00	0.00	
ITRF2005	2.6	1.0	-2.3	0.92	0.00	0.00	0.00	2010.0
rates	0.3	0.0	-0.1	0.03	0.00	0.00	0.00	
ITRF2000	0.7	1.2	-26.1	2.12	0.00	0.00	0.00	2010.0
rates	0.1	0.1	-1.9	0.11	0.00	0.00	0.00	
ITRF97	7.4	-0.5	-62.8	3.80	0.00	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	
ITRF96	7.4	-0.5	-62.8	3.80	0.00	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	
ITRF94	7.4	-0.5	-62.8	3.80	0.00	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	
ITRF93	-50.4	3.3	-60.2	4.29	-2.81	-3.38	0.40	2010.0
rates	-2.8	-0.1	-2.5	0.12	-0.11	-0.19	0.07	
ITRF92	15.4	1.5	-70.8	3.09	0.00	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	
ITRF91	27.4	15.5	-76.8	4.49	0.00	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	
ITRF90	25.4	11.5	-92.8	4.79	0.00	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	
ITRF89	30.4	35.5	-130.8	8.19	0.00	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	
ITRF88	25.4	-0.5	-154.8	11.29	0.10	0.00	0.26	2010.0
rates	0.1	-0.5	-3.3	0.12	0.00	0.00	0.02	

Transformation parameters from ITRF2014 to past ITRFs.

Note : These parameters are derived from those already published in the IERS Technical Notes and Annual Reports. The transformation parameters should be used with the standard model (1) given below and are valid at the indicated epoch.

[XS]	[X]	1	$\left[T_{x}\right]$		[D	$-R_z$	R_y] $[X]$	1
YS	= Y	+	T_y	+	R _z	D	$-R_x Y$	
$\lfloor zs \rfloor$	l_Z		T_z		$\left -R_{y}\right $	R_x	D] LZ	

(1)

(2)

Where X,Y,Z are the coordinates in ITRF2014 and XS,YS,ZS are the coordinates in the other frames.

On the other hand, for a given parameter P, its value at any epoch t is obtained by using equation (2).

P(t) = P(EPOCH) + P * (t - EPOCH)

where EPOCH is the epoch indicated in the above table (currently 2010.0) and P is the rate of that parameter.