

Algorithm Theoretical Basis Document
Land

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The approach is based on the simplified radiative transfer theory and the assumption of minimal atmospheric contribution. Assuming the canopy temperature is equal to the soil temperature and the heterogeneous mixtures of vegetation and roughness conditions within the satellite footprint are adequately represented by the footprint average. The radio-brightness temperature (TB) at a specific microwave frequency of a land surface can be represented by:

$$TB_{fp} = T\{(1 - r_{fp}) \exp(-\tau_{fp}) + (1 - \omega_{fp}) \times [1 - \exp(-\tau_{fp})] \times [1 + r_{fp} \exp(-\tau_{fp})]\} \quad (1)$$

where f is the frequency of the observation, p refers to horizontal (h) or vertical (v) polarization, T is effective surface temperature, r_{fp} is the soil reflectivity, ω_{fp} is the vegetation scattering albedo and τ_{fp} is the vegetation opacity along the observation path. Note that this expression assumes a constant incidence angle as employed in AMSR-E and AMSR2.

All algorithms, including this one, utilize the Fresnel equations (Jackson 1993, Owe et al. 2001, Wigneron et al. 2007, Gao et al. 2009) to relate the reflectivity (or emissivity) to the dielectric properties of the soil. The application of the Fresnel equations requires that the surface is smooth and that the contributing depth dielectric properties are uniform. The reflectivity in Eq. 1 is for the underlying soil, which may have a rough surface. This value must be related to a smooth surface equivalent. The surface roughness effect is typically modeled using the $h - Q$ formulation described by Wang and Choudhury (1995), where h_f and Q_f are parameters related to the surface height and horizontal roughness correlation length. Thus, if we assume that the single scattering albedo is negligible ($\omega \cong 0$), then Eq. 1 can be re-written so that TB_{fp} is directly related to smooth surface conditions.

$$TB_{fp} = T\{1 - [(1 - Q_f)r_{fp}^s + Q_f r_{fp}^{s-}] \times \exp(-h_f - 2\tau_{fp})\} \quad (2)$$

where r_{fp}^s is the smooth surface reflectivity.

The vegetation opacity can be expressed as a function of the vegetation water content (VWC) (Jackson and Schmugge 1991); the relationship between the two is given in Eq. 3.

$$\tau_{fp} = (b_{fp} \times VWC) / \cos \theta, \quad (3)$$

where b_{fp} is a vegetation attenuation parameter that is dependent on canopy type, polarization,

frequency, and θ is incidence angle.

Since both h_f and b_{fp} are known to increase with frequency and have similar attenuating effects (Eq. 2), it is possible to lump these together into a combined vegetation-roughness parameter. Njoku and Chan (2006) proposed that the frequency dependence of g could be approximated using a proportionality frequency dependent coefficient α_f . Consequently, Eq. 2 can be simplified further:

$$TB_{fp} = T[1 - r_{fp}^{s*} \times \exp(-\alpha_f g)], \quad (4)$$

where

$$r_{fp}^{s*} = (1 - Q_f)r_{fp}^s + Q_f r_{fq}^s \quad (5)$$

$$\alpha_f g = h_f + (2b_f \times VWC) / \cos \theta, \quad (6)$$

where p and q are h and v polarizations. Note that here we have assumed that the vegetation parameter is unpolarized, thus p was dropped.

It has been demonstrated that the vegetation and soil moisture conditions (SM) are both related to the microwave polarization difference ratio:

$$MPDI_f = (TB_{fv} - TB_{fh}) / (TB_{fv} + TB_{fh}) \quad (7)$$

This ratio is insensitive to surface temperature, which reduces the required number of sensor variables or the need for an ancillary data set.

Substituting 3 and 4 into 7 yields

$$MPDI_f = A_f (1 - 2Q_f) \{1 + B[\exp(\alpha_f g) - 1]\}^{-1} \quad (8)$$

$$A_f = [(1 - r_{fv}^s) - (1 - r_{fh}^s)] / [(1 - r_{fv}^s) + (1 - r_{fh}^s)]$$

$$B_f = 2 / [(1 - r_{fv}^s) - (1 - r_{fh}^s)]$$

A_f and B_f are both functions of soil moisture and A_f represents the $MPDI_f$ of the bare smooth soil.

As a result of the manipulations above, the soil moisture and vegetation/roughness dependences can be separated and the corresponding approximations are given in 9 and 10, respectively.

$$MPDI_f \approx A_f (1 - 2Q_f) \times \exp[-C_f B_f \alpha_f g] \quad (9)$$

$$MPDI_f \approx A_f(1 - 2Q_f) \times \exp[-D_f \alpha_f g], \quad (10)$$

where C_f and D_f are best-fit coefficients.

As defined earlier, g is a combined vegetation-roughness factor because both have similar impact on $MPDI_f$, i.e. an increase in h_f is manifested in g_f as an increase in vegetation water content. Thus, g can be interpreted as an equivalent vegetation water content in the units of kg/m^2 .

The first step in solving 9 and 10 is to determine the roughness parameters. Since h_f and Q_f have similar relationships with $MPDI_f$, there is some redundancy in varying both of these parameters. Therefore, h_f (incorporated in the $\alpha_f g$ term) was selected to represent the spatial variability, while Q_f is treated as a fixed global factor. Q_f was determined for each frequency by calibrating Eq. 8 to the AMSR-E computed $MPDI_f$ values over two desert regions, one located in Niger and the other one in Saudi Arabia. The radiative transfer simulations were carried out assuming bare, smooth, dry land surface conditions with $h_f = 0$ and $SM = 0.05 \text{ m}^3/\text{m}^3$. As computed, Q_f represents minimum roughness conditions. Spatial variations in surface roughness are then accounted for by allowing h_f to vary globally.

Frequency dependent coefficients C_f and D_f were determined next using a similar approach. Simulations results from the Dobson dielectric model (Dobson et al. 1985) for dry to moderate soil moisture ($SM=0.05-0.20 \text{ m}^3/\text{m}^3$) for sandy loam were used to estimate these parameters.

α_f , which was defined as a proportionality constant, allows us to account for the frequency dependence of the vegetation parameterization (Eq. 3). We already clarified that the canopy related b_f parameter in Eq. 3 is a function of vegetation type, thus, along with the frequency dependence α_f also needs to be calibrated over a wide range of vegetation conditions. This was done over a region of naturally varying vegetation and roughness with approximately uniform dry soil moisture that includes portions of Chad, Sudan, and the Central African Republic. AMSR-E observations for a dry month (March 2004) with a uniform value of $0.10 \text{ m}^3/\text{m}^3$ over this domain were used to estimate α_f .

g is then derived using Eq. 10. For a region of uniform soil moisture $MPDI_f$ approximated by

$$MPDI_f \approx A_f \times \exp[-\alpha_f g] \quad (11)$$

Njoku and Chan (2006) demonstrated that the ratio of Z_f for any two AMSR-E frequency pairs is approximately equal

$$Z_f = \ln[A_f(1 - 2Q_f)/MPDI_f] \quad (12)$$

The vegetation parameter (g) can be approximated using the $MPDI_f$ observations from 10.7 GHz and 18.0 GHz.

$$g = \beta_0 + \beta_1 \ln(MPDI_{10.7}) + \beta_2 \ln(MPDI_{18.7}), \quad (13)$$

where β_0 , β_1 , and β_2 are model constants and the $MPDI$ -subscript specifies the frequency.

Similarly, soil moisture can be expressed as a linear function of $MPDI_f$, where the slope of the fitted line is dependent on the local ground conditions as characterized by roughness and vegetation. The time varying soil moisture (SM^t) is estimated as a deviation from the annual minimum soil moisture conditions (SM^{dry}) observed at a particular location. Consequently, we can derive two regression models, the first solution using the current $MPDI_f$ value ($MPDI_{10.7}^t$) and the second one using an annual minimum baseline $MPDI$ for dry soil conditions ($MPDI_{10.7}^{dry}$). Subtracting the baseline from the current leads to the following equation:

$$SM^t - SM^{dry} = \alpha_o g^* + \alpha_1 (MPDI_{10.7}^t - MPDI_{10.7}^{dry}) \times \exp(\alpha_2 g^*), \quad (14)$$

where α_o , α_1 , and α_2 are model constants that incorporate the previously derived and calibrated α_f coefficients. g^* is computed using monthly minimum values of $MPDI_f$ at 10.7 GHz.

Finally, the algorithm relies on some additional ancillary data sets to identify areas of permanent ice, open water bodies or strong topography, where the retrieval is not carried out.

SCA Algorithm

In the single channel algorithm (SCA) (Jackson 1993), horizontally polarized TB_{hf} are traditionally used due to their sensitivity to soil moisture, but the same algorithm can also be applied to v polarization TB_{vf} . The use of h pol TB_{hf} with the SCA is the current SMAP baseline algorithm. In this approach, brightness temperatures are converted to emissivity using a surrogate for the physical temperature of the emitting layer. The derived emissivity is corrected for vegetation and surface roughness to obtain the soil emissivity, where. The Fresnel equation is then used to determine the dielectric constant. Finally, a dielectric mixing model is used to obtain the soil moisture. Additional details on these steps follow.

At the X-band frequency used by AMSR-E, the brightness temperature of the land surface is proportional to its emissivity (e_{fp}^{obs} , where $e_{fp}^{obs} = 1 - r_{fp}$) multiplied by its physical temperature (T). It is typically assumed that the temperatures of the soil and the vegetation are the same.

Based upon the above, the equation for observed TB_{fp} is:

$$TB_{fp} = e_{fp}^{obs} T + [1 - e_{fp}^{obs}] T_{fp}^{sky}, \quad (15)$$

where the second term in the equation represents the contributions of the cosmic background and downwelling radiation from the atmosphere as reflected by the soil surface. This term is very

small at L band and tends to be dropped for computational purposes (although it can be retained for stricter accuracy). Equation 1 can be rearranged to derive emissivity:

$$e_{fp}^{obs} = \frac{TB_{fp}}{T} \quad (16)$$

The physical temperature is estimated using Ka-band observations at v polarization (De Jeu and Owe 2003).

The emissivity retrieved above is that of the soil as modified by any overlying vegetation and surface roughness. In the presence of vegetation, the observed emissivity is a composite of the soil and vegetation. To retrieve soil water content, it is necessary to isolate the soil surface emissivity (e_{fp}^{surf}). Following Jackson and Schmugge (1991) the emissivity

$$e_{fp}^{obs} = [1 - \omega_{fp}][1 - \gamma_{fp}][1 + (1 - e_{fp}^{surf})\gamma_{fp}] + e_{fp}^{surf}\gamma_{fp} \quad (17)$$

Both the single scattering albedo (ω) and the one-way transmissivity of the canopy (γ) are dependent upon the vegetation structure, polarization and frequency. The transmissivity is a function of the optical depth (τ) of the vegetation canopy:

$$\gamma_{fp} = \exp[-\tau_{fp} \sec \theta] \quad (18)$$

The single scattering albedo tends to be very small, and sometimes is assumed to be zero in order to reduce dimensionality for computational purpose. Substituting equation 18 into equation 17 and rearranging yields

$$e_{fp}^{surf} = \frac{e_{fp}^{obs} - 1 + \gamma_{fp}^2 + \omega_{fp} - \omega_{fp}\gamma_{fp}^2}{\gamma_{fp}^2 + \omega_{fp}\gamma_{fp} - \omega_{fp}\gamma_{fp}^2} \quad (19)$$

The vegetation optical depth is also dependent upon the vegetation water content (VWC). In studies reported in Jackson and Schmugge (1991), it was found that the following functional relationship between the optical depth and vegetation water content could be applied Eq. 3.

The vegetation water content can be estimated using several ancillary data sources. The baseline approach utilizes a set of land cover-based equations to estimate VWC from values of the Normalized Difference Vegetation Index (NDVI) (an index derived from visible-near infrared reflectance data).

The emissivity that results from the vegetation correction is that of the soil surface, including any effects of surface roughness. These effects must be removed in order to determine the smooth surface soil emissivity (e_{fp}^{soil}) which is required for the Fresnel equation inversion. One approach to removing this effect is a model described in Choudhury et al. (1979) that yields the bare smooth soil emissivity:

$$e_{fp}^{soil} = 1 - [1 - e_{fp}^{surf}] \exp[-h_{fp} \cos^2 \theta] \quad (20)$$

The $\cos^2 \theta$ term is often dropped to avoid overcorrecting for roughness. The parameter h_{fp} is dependent on the polarization, frequency, and geometric properties of the soil surface.

Emissivity is related to the dielectric properties (ϵ_{fp}) of the soil and the viewing or incidence angle (θ). For ease of computational inversion, it is assumed that the real component (ϵ_{fp}^r) of the dielectric constant provides a good approximation of the complex dielectric constant; however, this assumption can be modified if additional evidence is found to support the use of this more complex formulation. The Fresnel equations link the dielectric constant to emissivity. For horizontal (h) polarization:

$$e_{hf}(\theta) = 1 - \left| \frac{\cos \theta - \sqrt{\epsilon_{hf}^r - \sin^2 \theta}}{\cos \theta + \sqrt{\epsilon_{hf}^r - \sin^2 \theta}} \right|^2 \quad (21)$$

The dielectric constant of soil is a composite of the values of its components – air, soil, and water, which have greatly different values. A dielectric mixing model is used to relate the estimated dielectric constant to the amount of soil moisture.

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