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ICE, CLOUD, and Land Elevation Satellite-2 (ICESat-2) Project

Algorithm Theoretical Basis Document (ATBD) For Land-Ice Along-Track Products Part 2: Land-ice H(t)/ATL11

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Abstract

CM Foreword

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Preface

This document is the Algorithm Theoretical Basis Document for the TBD processing to be implemented at the ICESat-2 Science Investigator-led Processing System (SIPS). The SIPS supports the ATLAS (Advance Topographic Laser Altimeter System) instrument on the ICESat-2 Spacecraft and encompasses the ATLAS Science Algorithm Software (ASAS) and the Scheduling and Data Management System (SDMS). The science algorithm software will produce Level 0 through Level 4 standard data products as well as the associated product quality assessments and metadata information.

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73 **Change History Log**

Revision Level	Description of Change	SCoRe No.	Date Approved
1.0	Initial Release		
1.1	Changes for release 002. Calculate all crossovers (including near 88 S), determine the center of the y_atc search from the median of unique pair center locations.		
1.2	Changes for release 003. Add geoid and dem parameters.		
1.3	Improved the description of the polynomial coefficient writing process		

List of TBDs/TBRs

Item No.	Location	Summary	Ind./Org.	Due Date

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1.0 INTRODUCTION

This document describes the theoretical basis and implementation of the level-3b land-ice processing algorithm for ATL11, which provides time series of surface heights. The higher-level products, providing gridded height, and gridded height change will be described in supplements to this document available in early 2020.

ATL11 is based on the ICESat-2 ATL06 Land-ice Height product, which is described elsewhere(Smith and others, 2019a, Smith and others, 2019b). ATL06 provides height estimates for 40-meter overlapping surface segments, whose centers are spaced 20 meters along each of ICESat-2's RPTs (reference pair tracks), but displaced horizontally both relative to the RPT and relative to one another because of small (a few tens of meters or less) imprecisions in the satellite's control of the measurement locations on the ground. ATL11 provides heights corrected for these offsets between the reference tracks and the location of the ATLAS measurements. It is intended as an input for high-level products, ATL15 and ATL16, which will provide gridded estimates of ice-sheet height and height change, but also may be used alone, as a spatially-organized product that allows easy access to height-change information derived from ICESat-2.

ATL11 employs a technique which builds upon those previously used to measured short-term elevation changes using ICESat repeat-track data. Where surface slopes are small and the geophysical signals are large compared to background processes (i.e., ice plains and ice shelves), some studies have subtracted the mean from a collection of height measurements from the same repeat track to leave the rapidly-changing components associated with subglacial water motion (Fricker and others, 2007) or tidal flexure (Brunt and others, 2011). In regions where off-track surface slopes are not negligible, height changes can be recovered if the mean height and an estimate of the surface slope (Smith and others, 2009) are subtracted from the data, although in these regions the degree to which the surface slope estimate and the elevation-change pattern are independent is challenging to quantify.

ICESat-2's ATL06 product provides both surface height and surface-slope information each time it overflies its reference tracks. The resulting data are similar to that from the scanning laser altimeters that have been deployed on aircraft in Greenland and Antarctica for two decades (cite), making algorithms originally developed for these instruments appropriate for use in interpreting ATLAS data. One example is the SERAC (Surface Elevation Reconstruction and Change Detection) algorithm (Schenk & Csatho, 2012) provides an integrated framework for the derivation of elevation change from altimetry data. In SERAC, polynomial surfaces are fit to collections of altimetry data in small (< 1 km) patches, and these surfaces are used to correct the data for sub-kilometer surface topography. The residuals to the surface then give the pattern of elevation change, and polynomial fits to the residuals as a function of time give the long-term pattern of elevation change. The ATL11 algorithm is similar to SERAC, except that (1) polynomial fit correction is formulated somewhat differently, so that the ATL11 correction gives the surface height at the fit center, not the height residual, and (2) ATL11 does not include a polynomial fit with respect to time.

2.0 BACKGROUND INFORMATION AND OVERVIEW

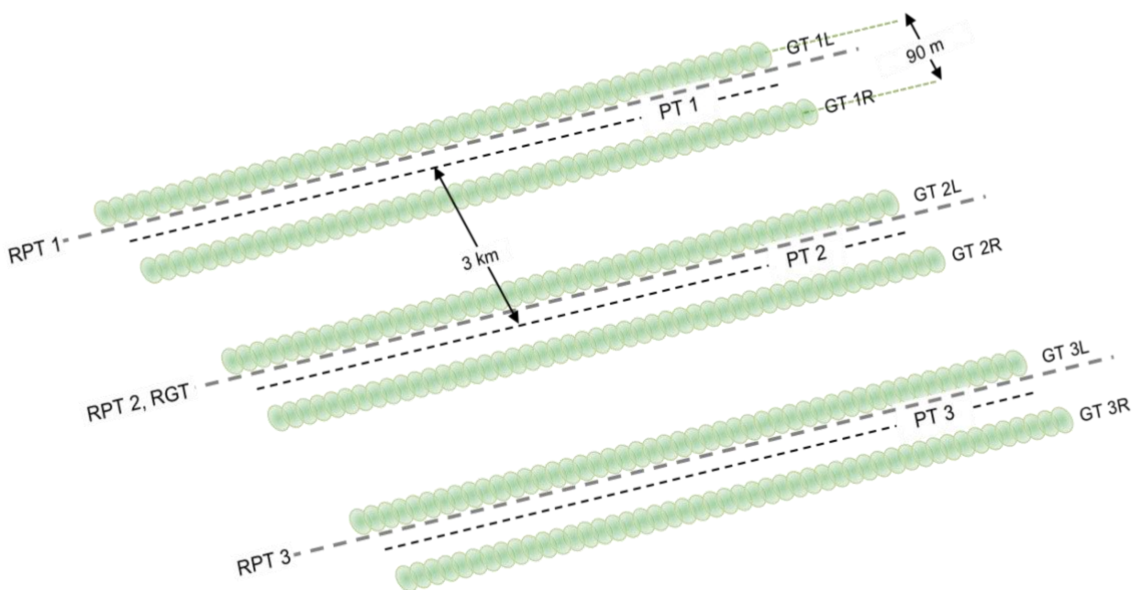
This section provides a conceptual description of ICESat-2's ice-sheet height measurements and gives a brief description of the derived products.

2.1 Background

The primary goal of the ICESat-2 mission is to estimate mass-balance rates for the Earth's ice sheets. An important step in this process is the calculation of height change at specific locations on the ice sheets. In an ideal world, a satellite altimeter would exactly measure the same point on the earth on each cycle of its orbit. However, there are limitations in a spacecraft's ability to exactly repeat the same orbit and to point to the same location. These capabilities are greatly improving with technological advances but still have limits that need to be accounted for when estimating precise elevation changes from satellite altimetry data. The first ICESat mission allowed estimates of longer-term elevation rates using along-track differencing, because ICESat's relatively precise (50-150-m) pointing accuracy, precise (4-15 m) geolocation accuracy, and small (35-70-m) footprints allowed it to resolve small-scale ice-sheet topography. However, because ICESat had a single-beam instrument, its repeat-track measurements were reliable only for measuring the mean rate of elevation change, because shorter-term height differences could be influenced by the horizontal dispersion of tracks on a sloping surface. ICESat-2 makes repeat measurements over a set of 1387 reference ground tracks (RGTs), completing a *cycle* over all of these tracks every 91 days. ICESat-2's ATLAS instrument employs a split-beam design, where each laser pulse is divided six separate beams. The beams are organized into three *beam pairs*, with each separated from its neighbors by 3.3 km (**Figure** 2-1), each pair following a reference pair track (RPT) that is parallel to the RGT. The beams within each pair separated by 90 m, which means that each cycle's measurement over an RPT can determine the surface slope independently, and a height difference can be derived from any two measurements of an RPT. The 90-m spacing between the laser beams in each pair is equal to twice the required RMS accuracy with which ICESat-2 can be pointed at its RPTs, which means that for most, but not all, repeat measurements of a given RPT, the pairs of beams will overlap one another. To obtain a record of elevation change from the collection of paired measurements on each RPT, some correction is still necessary to account for the effects of small-scale surface topography around the RPT in the ATL06 surface heights that appear as a result of this non-exact pointing. ATL11 uses a polynomial fit to the ATL06 measurements to correct for small-scale topography effects on surface heights that result from this non-exact pointing.

The accuracy of ICESat-2 measurements depends on the thickness of clouds between the satellite and the surface, on the reflectance, slope, and roughness of the surface, and on background noise rate which, in turn, depends on the intensity of solar illumination of the surface and the surface reflectance. It also varies from laser beam to beam, because in each of ICESat-2's beam pairs one beam (the "strong beam") has approximately four times the signal strength of the other (the "weak beam"). Parameters on the ATL06 product allow estimation of errors in each measurement, and allow filtering of most measurements with large errors due to misidentification of clouds or noise as surface returns (blunders), but to enable higher precision surface change estimates, ATL11 implements further self-consistency checks that further reduce the effects of errors and blunders.

Figure 2-1. ICESat-2 repeat-track schematic



Schematic drawing showing the pattern made by ATLAS's 6-beam configuration on the ground, for a track running from lower left to upper right. The 6 beams are grouped into 3 beam pairs with a separation between beams within a pair of 90m and a separation between beam pairs of 3.3 km. The RPTs (Reference Pair Tracks, heavily dashed lines in gray) are defined in advance of launch; the central RPT follows the RGT (Reference Ground Track, matching the nadir track of the predicted orbit). The Ground Tracks are the tracks actually measured by ATLAS (GT1L, GT1R, etc., shown by green footprints). Measured Pair Tracks (PTs, smaller dashed lines in black) are defined by the centers of the pairs of GTs, and deviate slightly from the RPTs because of inaccuracies in repeat-track pointing. The separation of GTs in each pair in this figure is greatly exaggerated relative to the separation of the PTs.

2.2 Elevation-correction Coordinate Systems

We perform ATL11 calculations using the along-track coordinate system described in the ATL06 ATBD (Smith and others, 2019b, Smith and others, 2019a). The along-track coordinate is measured parallel to the RGT, starting at each RGT's origin at the equator. The across-track coordinate is measured to the left of the RGT, so that the two horizontal basis vectors and the local vertical vector form a right-handed coordinate system.

2.3 Terminology:

Some of the terms that we will use in describing the ATL11 fitting process and the data contributing are:

RPT: Reference pair track

Cycle: ICESat-2 has 1387 distinct reference ground tracks, which its orbit covers every 91 days. One repeat measurement of these reference ground tracks constitutes a cycle.

ATL06 segment: A 40-meter segment fit to a collection of ATL03 photon-event data, as described in the ATL06 ATBD

ATL06 pair: Two ATL06 segments from the same cycle with the same *segment_id*. By construction, both segments in the ATL06 pair have the same along-track coordinate, and are separated by the beam-to-beam spacing (approximately 90 m) in the across-track direction

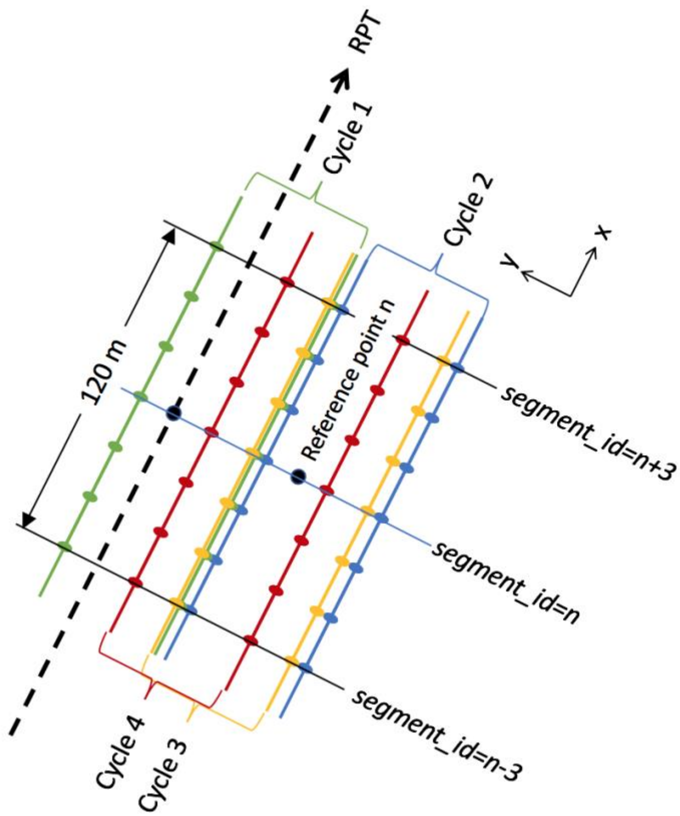
ATL11 RPT point: The expected location of each ATL11 point on the RPT, equivalent to the beginning of every third geosegment on the RPT, or the center of every third ATL06 segment.

ATL11 reference point: an *ATL11 RPT point* shifted in the across-track direction to better match the geometry of the available ATL06 data.

ATL11 fit: The data and parameters associated with a single ATL11 reference point. This includes corrected heights from all available cycles

ATL11 calculates elevations and elevation differences based on collections of segments from the same beam pair but from different cycles. ATL11 is posted every 60 m, which corresponds to every third ATL06 *segment_id*, and includes ATL06 segments spanning three segments before and after the central segment, so that the ATL11 uses data that span 120 m in the along-track direction. ATL11 data are centered on *reference points*, which has the same along-track coordinate as its central ATL06 segment, but is displaced in the across-track direction to better match the locations of the ATL06 measurements from all of the cycles present (see section 3.1.3).

Figure 2-2. ATL06 data for an ATL11 reference point



Schematic of ATL06 data for an ATL11 reference point centered on segment n, based on data from four cycles. The segment centers span 120 m in the along-track data, and the cycles are randomly displaced from the RPT in the across-track direction. The reference point has an along-track location that matches that of segment n, and an across-track position chosen to match the displacements of the cycles.

2.4 Repeat and non-repeat cycles in the ICESat-2 mission

In the early part of the ICESat-2 mission, an error in the configuration of the start trackers prevented the instrument from pointing precisely at the RGTs. As a result, all data from cycles 1 and 2 were measured between one and two kilometers away from the RGTs, with offsets that varied in time and as a function of latitude. The measurements from cycles 1 and 2 still give high-precision measurements of surface height, but repeat-track measurements from ICESat-2 begin during cycle 3, in April of 2019. ATL11 files will be generated for ATL06 granules from cycles 1 and 2, but these will contain only one cycle of data, plus crossovers, because the measurements from these cycles (which are displaced from the RPTs by several kilometers) will not be repeated. We expect the measurements from cycles 1 and 2 to be useful as a reduced-resolution (compared to ATL06) mapping of the ice sheet, which may prove useful in DEM generation and in comparisons with other altimetry missions. For cycles 3 and after, each ATL11 granule will contain all available cycles for each RGT (i.e. from cycle 3 onwards), and will contain crossovers between the repeat cycles and cycles 1 and 2.

Outside the polar regions, ICESat-2 is pointed to minimize gaps between repeat measurements, and so does not make repeat measurements over its ground tracks. ATL11 is only calculated within the repeat-pointing mask (see Figure ???), which covers areas poleward of 60°N and 60°S.

2.5 Physical Basis of Measurements / Summary of Processing

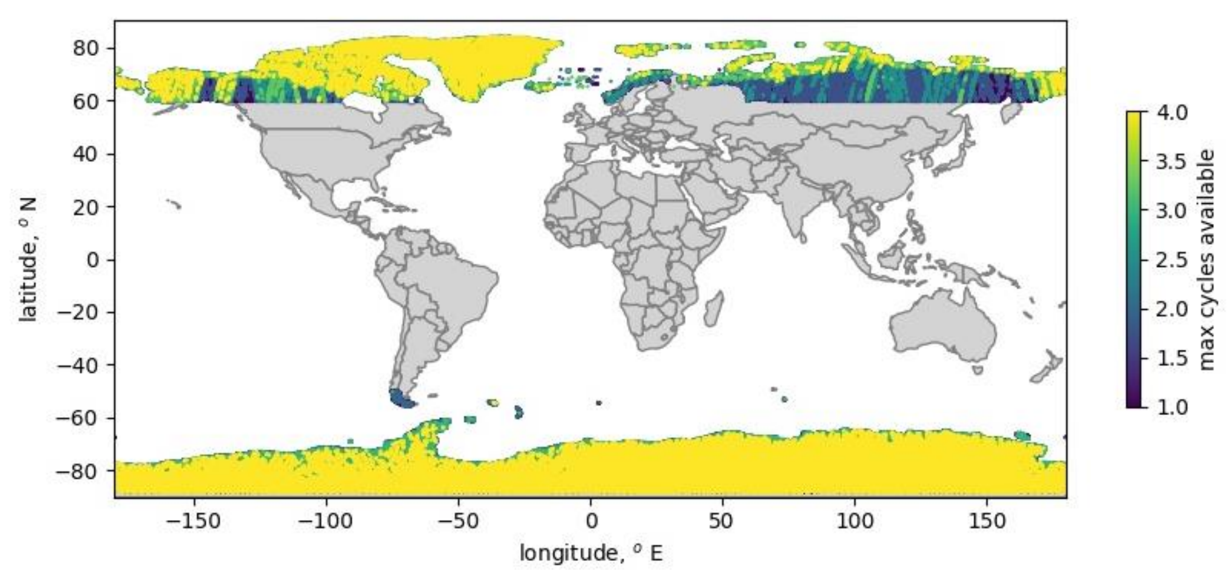
Surface slopes on the Antarctic and Greenland ice sheets are generally small, with magnitudes less than two degrees over 99% of Antarctica's area. Smaller-scale (0.5-3 km) undulations, generated by ice flow over hilly or mountainous terrain may have amplitudes of up to a few degrees. Although we expect that the surface height will change over time, slopes and locations of these smaller-scale undulation are likely controlled by underlying topography and should remain essentially constant over periods of time comparable with the expected 3-7 duration of the ICESat-2 mission. This allows us to use estimates of ice-sheet surface shape derived from data spanning the full mission to correct for small (<130-m) differences in measurement locations between repeat measurements of the same RPT, to produce records of height change for specific locations. To account for changes in the ice-sheet surface slope associated with gradients in thinning, we also solve for the rate of surface-slope change, when sufficient data are available. Further, we can use the surface slope estimates in ATL06 to determine whether different sets of measurements for the same fit center are self-consistent: We can assume that if an ATL06 segment shows a slope significantly different from others measured near the same reference point it likely is in error. The combination of parameters from ATL06 and these self-consistency checks allows us to generate time series based on the highest-quality measurements for each reference point, and our reference surface calculation lets us correct for small-scale topography and to estimate error magnitudes in the corrected data.

2.5.1 Choices of product dimensions

We have chosen a set of dimensions for the ATL11 fitting process with the goal of creating a product that is conveniently sized for analysis of elevation changes, while still capturing the details of elevation change in outlet glaciers. The assumption that ice-sheet surface can be approximated by a low-degree polynomial becomes untenable as data from larger and larger areas are included in the calculation; therefore we use data from the smallest feasible area to define our reference surface, while still including enough data to reduce the sampling error in the data and to allow for the possibility that at least one or two will encounter a flat surface, which greatly improves the chances that each cycle will be able to measure surface comparable to one another. Each ATL11 point uses data from an area up to 120 m in the along-track direction by up to 130 m in the across-track direction. We have chosen the cross-track search distance (L_{search_XT}) to be 65 m, approximately equal to half the beam spacing, plus three times the observed 6.5 m standard deviation of the across-track pointing accuracy for cycles 3 and 4 in Antarctica. We chose the across-track search distance (L_{search_AT}) to be 60 m, approximately equal to L_{search_XT}, so that the full L_{search_AT} search window spans three ATL06 segments before and after the central segment for each reference point. The resulting along-track resolution is around one third that of ATL06, but still allows 6-7 distinct elevation-change samples across a small (1-km) outlet glacier.

2.6 Product coverage

Figure 2-3. Potential ATL11 coverage



Maximum number of valid repeat measurements from an ATL11 file for each 10-km segment of pair track 2. Yellower colors indicate areas where ICESat-2 has systematically pointed at the RGTs.

Over the vegetated parts of the Earth, ICESat-2 makes spatially dense measurements, measuring tracks parallel to the reference tracks in a strategy that will eventually measure global vegetation with a track-to-track spacing better than 1 km. Because ATL11 relies upon repeat measurements over reference tracks to allow the calculation of its reference surfaces, ATL11 is generated for ICESat-2 subregions 3-5 and 10-12 (global coverage, north and south of 60 degrees). Repeat measurements are limited to Antarctica, Greenland, and the High Arctic islands (Figure 2-3), although in other areas the fill-in strategy developed for vegetation measurements allows some repeat measurements. In regions where ICESat-2 was not pointed to the repeat track, most ATL11 reference points will provide one measurement close to the RPT. Crossover data are available for many of these points, though their distribution in time is not regular. A future update to the product may provide crossover measurements for lower-latitude areas, but the current product format is not designed to allow this.

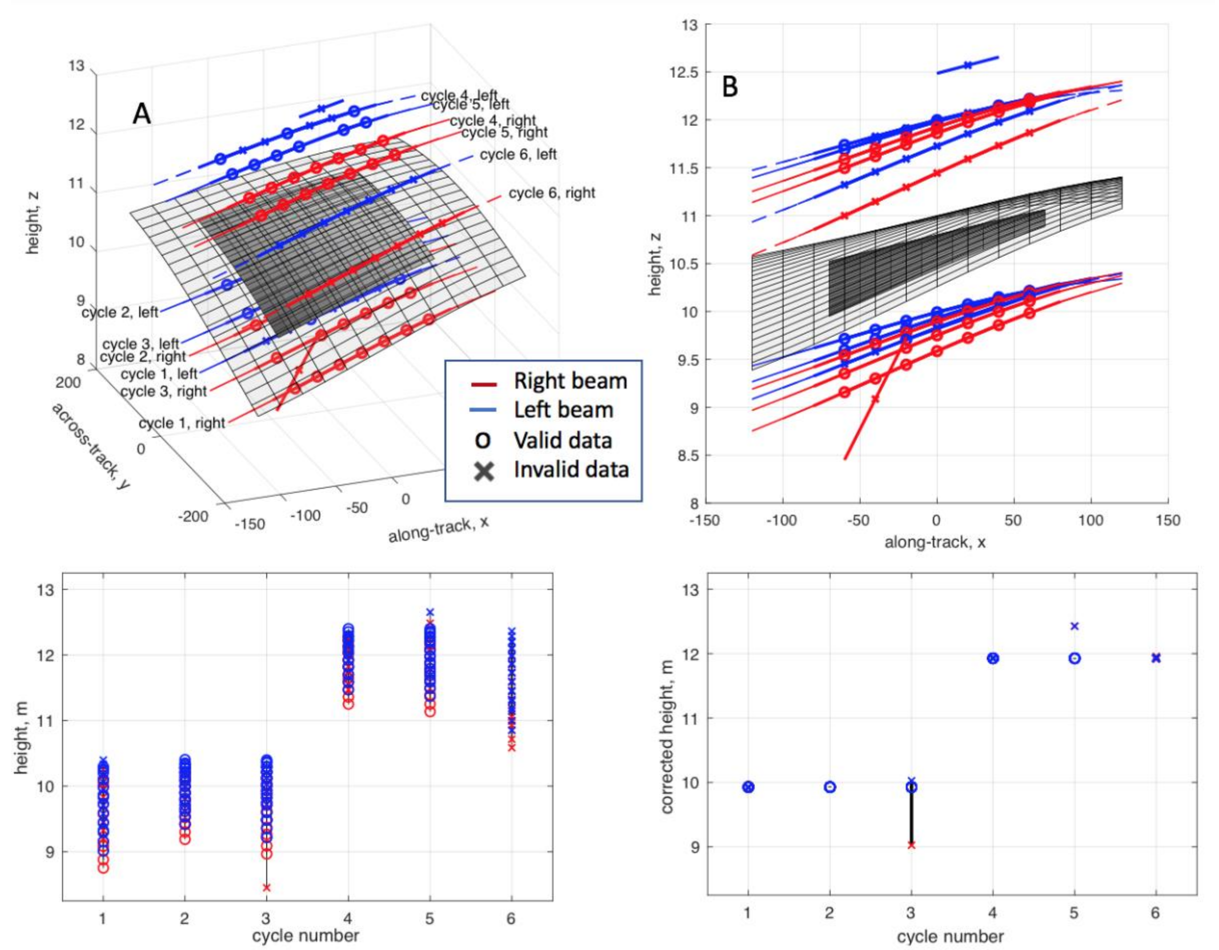
3.0 ALGORITHM THEORY: DERIVATION OF LAND ICE H (T)/ATL11 (L3B)

In this section, we describe in detail the algorithms used in calculating the ATL11 land-ice parameters. This product is intended to provide time series of surface heights for land-ice and ice-shelf locations where ICESat-2 operates in repeat-track mode (*i.e.* for polar ice), along with parameters useful in determining whether each height estimate is valid or a result of a variety of potential errors (see ATL06 ATBD, section 1).

ATL11 height estimates are generated by correcting ATL06 height measurements for the combined effects of short-scale (40-120-m) surface topography around the fit centers and small (up to 130-m) horizontal offsets between repeat measurements. We fit a polynomial reference surface to height measurements from different cycles as a function of horizontal coordinates around the fit centers, and use this polynomial surface to correct the height measurements to the fit center. The resulting values reflect the time history of surface heights at the reference points, with minimal contributions from small-scale local topography.

In this algorithm, for a set of reference points spaced every 60 meters along each RPT (centered on every third segment center), we consider all ATL06 segments with centers within 60 m along-track and 65 m across-track of the reference point, so that each ATL11 fit contains as many as seven distinct along-track segments from each laser beam and cycle. We select a subset of these segments with consistent ATL06 slope estimates and small error estimates, and use these segments to define a time-variable surface height and a polynomial surface-shape model. We then use the surface-shape model to calculate corrected heights for the segments from cycles not included in the initial subset. We propagate errors for each of these steps to give formal errors estimates that take into account the sampling error from ATL06, and propagate the geolocation errors with the slope of the surface-shape model to give an estimate of systematic errors in the height estimates.

Figure 3-1. ATL11 fitting schematic



Schematic of the ATL11 fitting strategy. A and B show different renderings of the same set of data, A in perspective view and B from along the y (along-track axis). Lines show simulated ATL11 profiles; symbols show segment centers for segments within 60 m of the fit center (at x=y=0). Red lines and symbols indicate left beams, blue indicate right beams. 'o' markers indicate valid data segments, 'x' markers indicate invalid data segments. We plot the unperturbed, true surface height as a light-colored semi-transparent mesh, and the recovered surface height as a gray-shaded, opaque surface, shifted vertically to match the true surface. The gray surface shows the fit correction surface, offset vertically to match the true surface. C shows the uncorrected heights as a function of cycle number, and D shows the corrected heights (bottom), plotted for each repeat.

Figure 3-1 shows a schematic diagram of the fitting process. In this example, we show simulated ATL06 height measurements for six 91-day orbital cycles over a smooth ice-sheet surface (transparent grid). Between cycles 3 and 4, the surface height has risen by 2 m. Two of the segments contain errors: The weak beam for one segment from repeat 3 is displaced downward and has an abnormal apparent slope in the *x* direction, and one segment from repeat 5 is displaced upwards, so that its pair has an abnormal apparent slope in the *y* direction. Segments falling within the across and along-track windows of the reference point (at *x*=*y*=0 in this plot) are selected, and fit with a polynomial reference surface (shown in gray). When plotted as a function of cycle number (panel C), the measured heights show considerable scatter but when corrected to the reference surface (panel D), each cycle shows a consistent height, and the segments with errors are clearly distinct from the accurate measurements.

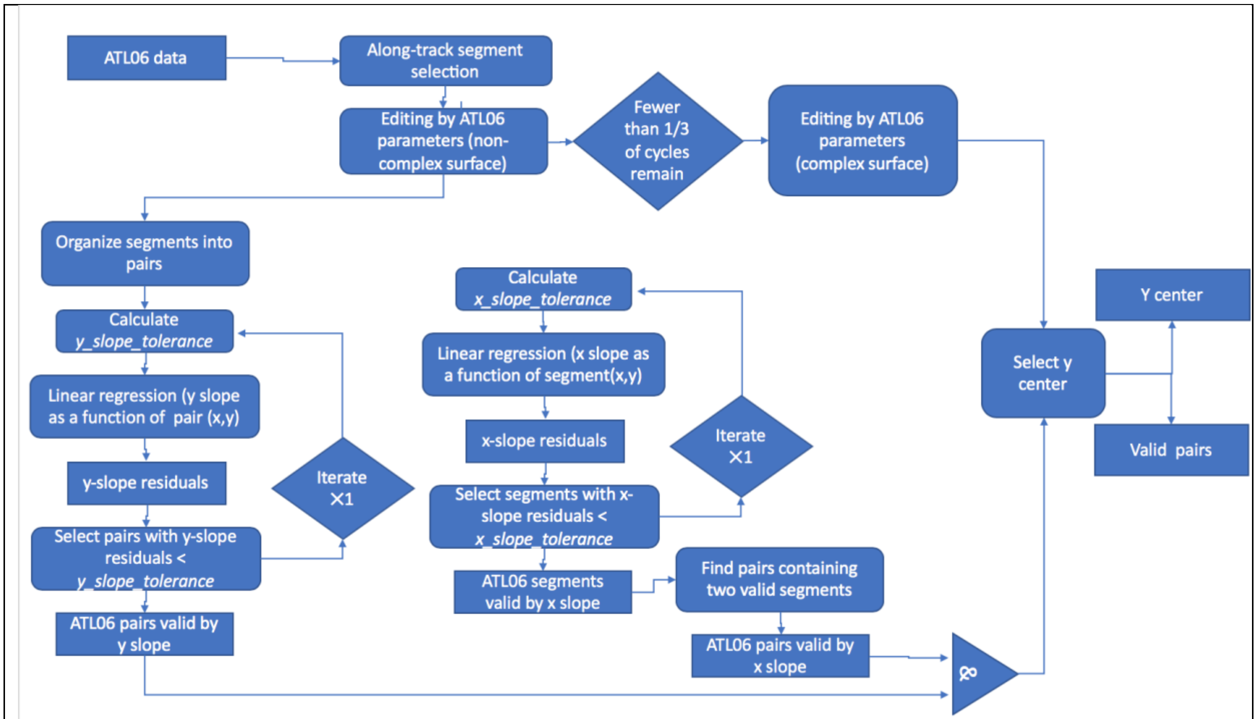
3.1 Input data editing

Each ATL06 measurement includes location estimates, along- and across-track slope estimates, and PE (Photon-Event)-height misfit estimates. To calculate the reference surface using the most reliable subset of available data, we perform tests on the surface-slope estimates and error statistics from each ATL06-pair to select a self-consistent set of data. These tests determine whether each pair of measurements is *valid* and can be used in the reference-shape calculation or is *invalid*. Segments from invalid pairs may be used in elevation-change calculations, but not in the reference-shape calculation.
A complete flow chart of the data-selection process is shown in Figure 3-2, and the parameters used to make these selections and their values are listed in Table 3-1.

Table 3-1 Parameter Filters to determine the validity of segments for ATL11 estimates

complex_surface_flag	**Segment parameter**	**Filter strategy**	**Section**
0	*ATL06_quality_summary*	*ATL06_quality_summary* =0 (indicates high-quality segments)	*3.1.1*
1	*SNR_significance*	*SNR_significance < 0.02* (indicates low probability of surface-detection blunders)	*3.1.1*
0 or 1	Along-track differences	Minimum height difference between the endpoints of a segment and the middles of its neighbors must be < 2 m (for smooth surfaces) or < 10 m (for complex surfaces)	*3.1.1*
0 or 1	*h_li_sigma*	*h_li _sigma* < max(0.05, 3*median(*h_li_sigma*))	3.1.1
0 or 1	Along-track slope	|*r_slope_x*|< 3 *slope_tolerance_x*	3.1.2
0 or 1	Across-track slope	|*r_slope_y*| < 3 *slope_tolerance_y*	3.1.2
0 or 1	Segment location	$\|x\_atc-x_0\| < L\_search\_XT$ $\|y\_atc-y_0\| < L\_search\_AT$	3.1.3

Figure 3-2. Data selection



Flow chart showing steps in the data selection process (Section 3.1)

3.1.1 Input data editing using ATL06 parameters

For each reference point, we collect all ATL06 data from all available repeat cycles that have *segment_id* values within ±3 of the reference point (inclusive) and that are on the same *rgt* and pair track as the reference point. The *segment_id* criterion ensures that the segment centers are within ±60 m of the reference point in the along-track direction. We next check that the ATL06 data are close to the pre-defined reference track, by rejecting all ATL06 segments that are more than 500 m away from the nominal pair across-track coordinates (-3200, 0, and 3200 meters for right, center, and left pairs, respectively). This removes data that were intentionally or accidentally collected with ATLAS pointed off nadir (i.e. for calibration scan maneuvers). ATL06 contains some segments with signal-finding blunders (Smith et al., 2019). To avoid having these erroneous segments contaminate ATL11, we filter using one of two sets of tests, depending on surface roughness. We identify high-quality ATL06 segments, using parameters that depend on whether the surface is identified as smooth or rough, as follows:
1) For smooth ice-sheet surfaces, we use the ATL06 *ATL06_quality_summary* parameter, combined with a measure of along-track elevation consistency, *at_min_dh*, that is calculated as part of ATL11. *ATL06_quality_summary* is based on the spread of the residuals for each segment, the along-track surface slope, the estimated error, and the signal strength. Zero values indicate that no error has been found. We define the along-track consistency parameter *at_min_dh* as the minimum absolute difference between the heights of the endpoints of each segment and the center heights of the previous and subsequent segments. Its value will be small if a segment's height and slope are consistent with at least one of its neighbors. For smooth

surfaces, we require that the *at_min_dh* values be less than 2 m. Over smooth ice-sheet surfaces, the 2-m threshold eliminates most blunders without eliminating a substantial number of high-quality data points.

2) For rough, crevassed surfaces, the smooth-ice strategy may not identify a sufficient number of pairs for ATL11 processing to continue. If fewer than one third of the original cycles remain after the smooth-surface criteria are applied, we relax our criteria, using the signal-to-noise ratio (based on the ATL06 *segment_stats/snr_significance* parameter) to select the pairs to include in the fit, and require that the *at_min_dh* values be less than 10 m. If we relax the criteria in this way, we mark the reference point as having a complex surface using the *ref_surf/complex_surface_flag,* which limits the degree of the polynomial used in the reference surface fitting to 0 or 1 in each direction.

For either smooth or rough surfaces, we perform an additional check using the magnitude of *h_li_sigma* for each segment. If any segment's value is larger than three times the maximum of 0.05 m and the median *h_li_sigma* for the valid segments for the current reference point, it is marked as invalid. The limiting 0.05 m value prevents this test from removing high-quality data over smooth ice-sheet surfaces, where errors are usually small.

Each of these tests applies to values associated with ATL06 segments. When the tests are complete, we check each ATL06 pair (*i.e.* two segments for the same along-track location from the same cycle) and if either of its two segments has been marked as invalid, the entire pair is marked as invalid.

3.1.2 Input data editing by slope

The segments selected in 3.1.1 may include some high-quality segments and some lower-quality segments that were not successfully eliminated by the data-editing criteria. We expect that the ATL06 slope fields (*dh_fit_dx*, and *dh_fit_dy*) for the higher-quality data should reflect the shape of an ice-sheet surface with a spatially consistent surface slope around each reference point, but that at least some of lower-quality data should have slope fields that outliers relative to this consistent surface slope. In this step, we assume that the slope may vary linearly in *x* and *y*, and so use residuals between the slope values and a regression of the slope values against *x* and *y* to identify the data with inconsistent slope values. The data with large residuals are marked as *invalid.*

Starting with valid pairs from 3.2.1, we first perform a linear regression between the *y* slopes of the pairs and the pair-center *x* and *y* positions. The residuals to this regression define one *y_slope_residual* for each pair. We compare these residuals against a *y_slope_tolerance:*

$$y\_slope\_tolerance = max(0.01, 3\ median\ (dh\_fit\_dy\_sigma), 3\ RDE\ (y\_slope\_residuals)) \qquad 1$$

Here RDE is the Robust Difference Estimator, equal to half the difference between the 16th and 84th percentiles of a distribution, and the minimum value of 0.01 ensures that this test does not remove high-quality segments in regions where the residuals are very consistent. If any pairs have a *y_slope_residual* greater than *y_slope_tolerance,* we remove them from the group of valid pairs, then repeat the regression, recalculate *y_slope_tolerance,* and retest the remaining pairs.

We then return to the pairs marked as *valid* from 3.1.1, and perform a linear regression between the *x* slopes of the segments within the pairs and the segment-center *x* and *y* positions. The residuals to this regression define one *x_slope_residual* for each segment. We compare these residuals against an *x_slope_tolerance*, calculated in the same way as (1), except using segment *x* slopes and residuals instead of pair *y* slopes. As with the *y* regression, we repeat this procedure once if any segments are eliminated in the first round.

After both the *x* and *y* regression procedures are complete, each pair of segments is marked as *valid* if both of its *x* residuals are smaller than *slope_tolerance_x* and its *y* residual is smaller than *slope_tolerance_y*.

3.1.3 Spatial data editing

The data included in the reference-surface fit fall in a "window" defined by a $2L_{search\ XT}$ by $2L_{search\ AT}$ rectangle, centered on each reference point. Because the across-track location of the repeat measurements for each reference point are determined by the errors in the repeat track pointing of ATLAS, a data selection window centered on the RPT in the *y* direction will not necessarily capture all of the available cycles of data. To improve the overlap between the window and the data, we shift the reference point in the *y* direction so that the window includes as many valid beam pairs as possible. We make this selection after the parameter-based (3.1.1) and slope-based (3.1.2) editing steps because we want to maximize the number high-quality pairs included, without letting the locations of low-quality segments influence our choice of the reference-point shift.

We select the across-track offset for each reference point by searching a range of offset values, δ, around the RPT to maximize the following metric:

$$M(\delta) = [number\ of\ unique\ valid\ pairs\ entirely\ contained\ in\ \ \delta \pm L_{search\ XT}] + [number\ of\ unpaired\ segments\ contained\ in\ \ \delta \pm L_{search\ XT}]/100 \quad (2)$$

Maximizing this metric allows the maximum number of pairs with two valid segments to be included in the fit, while also maximizing the number of segments included close to the center of the fit. If multiple values of δ have the same M value we choose the median of those δ values. The across-track coordinate of the adjusted reference point is then $y_0+\delta_{max}$, where y_0 is the across-track coordinate of the unperturbed reference point. After this adjustment, the segments in pairs that are contained entirely in the across-track interval $\delta \pm L_{search\ XT}$ are identified as *valid* based on the spatial search.

The location of the adjusted reference point is reported in the data group for each pair track, with corresponding local coordinates in the *ref_surf* subgroup: */ptx/ref_surf/x_atc*, */ptx/ref_surf/y_atc*.

3.2 Reference-Surface Shape Correction

To calculate the reference-surface shape correction, we construct the background surface shape from valid segments selected during 3.1 and 3.2, using a least-squares inversion that separates surface-shape information from elevation-change information. This produces surface shape-corrected height estimates for cycles containing at least one valid pair, and a surface-shape model that we use in later steps (3.4, 3.6) to calculate corrected heights for cycles that contain no valid pairs and to calculate corrected heights for crossing tracks.

3.2.1 Reference-surface shape inversion

The reference-shape inversion solves for a reference surface and a set of corrected-height values that represent the time-varying surface height at the reference point. The inversion involves three matrices:
(*i*): a polynomial surface shape matrix, **S**, that describes the functional basis for the spatial part of the inversion:

$$\mathbf{S} = \left[\left(\frac{x - x_0}{l_0}\right)^p \left(\frac{y - y_0}{l_0}\right)^q\right] \tag{3}$$

Here x_0 and y_0 are equal to the along-track coordinates of the adjusted reference point, */ptx/ref_surf/x_atc* and */ptx/ref_surf/y_atc*, respectively. **S** has one column for each permutation of p and q between zero and the degree of the surface polynomial in each dimension, but does not include a $p=q=0$ term. The degree is chosen to be no more than 3 (in the along-track direction) or 2 (in the across-track direction), and to be no more than the number of distinct pair-center y values (in the across-track direction) or more than 1 less than the number of distinct x values (in the along-track direction) in any cycle, with distinct values defined at a resolution of 20 m in each direction. The scaling factor, l_0, ensures that the components of S are on the order of 1, which improves the numerical accuracy of the computation. We set l_0=100 m, to approximately match the intra-pair beam spacing.
(ii): a matrix that encodes the repeat structure of the data, that accounts for the height-change component of the inversion:

$$\mathbf{D} = [\delta(i,1), \delta(i,2), \ \ \dots, \delta(i,N)\ ] \tag{4}$$

Here δ is the delta function, equal to 1 when its arguments are equal, zero otherwise, and i is an index that increments by one for each distinct cycle in the selected data.
(iii): a matrix that describes the linear rate of change in the surface slope over the course of the mission:

$$\mathbf{S}_t = \left[\left(\frac{x - x_0}{l_0}\right)\left(\frac{t - t_0}{\tau}\right), \left(\frac{y - y_0}{l_0}\right)\left(\frac{t - t_0}{\tau}\right)\right] \tag{5}$$

Here t_0 is equal to *slope_change_t0*, the mid-point of the mission at the time that ATL11 is generated, halfway between start repeat track pointing (the beginning of cycle 3) and either the end of the mission or the processing time (*slope_change_t0 is an attribute of each ATL11 file)*. This implies that on average, $(t - t_0)$ will have a zero mean. The time-scaling factor, τ, is equal to one year (86400*365.25 seconds). This component will only be included in ATL11 once eight complete cycles of data are available on the RGTs (after cycle 10 of the mission).
The surface shape, slope change, and height time series are estimated by forming a composite design matrix, **G**, where

$$\mathbf{G} = [\mathbf{S}\ \mathbf{S}_t\ \mathbf{D}], \tag{6}$$

and a covariance matrix, **C**, containing the squares of the segment-height error estimates on its diagonal. The surface-shape polynomial and the height changes are found:

$$[\boldsymbol{s}, \boldsymbol{s}_t, \boldsymbol{z}_c] = \mathbf{G}^{-\mathbf{g}}\boldsymbol{z}$$

where $\qquad (7)$

$$\mathbf{G}^{-\mathbf{g}} = [\mathbf{G}^T\mathbf{C}^{-1}\mathbf{G}]^{-1}\mathbf{G}^T\mathbf{C}^{-1}$$

The notation $[]^{-1}$ designates the inverse of the quantity in brackets, and **z** is the vector of segment heights. The parameters derived in this fit are **s**, a vector of surface-shape polynomial coefficients, $\mathbf{s}_t$, the mean rate of surface-slope change, and $\mathbf{z}_c$, a vector of corrected height values, giving the height at (lat_0, lon_0) as inferred from the height measurements and the surface polynomial. The matrix $\mathbf{G}^{-g}$ is the generalized inverse of **G**. The values of **s** are reported in the *ref_surf/poly_ref_surf* parameter, as they are calculated from (6), with no correction made for the scaling in (3). The values for the slope-change rates are reported in *ref_surf/slope_change_rate*, after rescaling to units of $years^{-1}$.

3.2.2 Misfit analysis and iterative editing

If blunders remain in the data input to the reference-surface calculation, they can lead to inaccurate reference surfaces. To help remove these blunders, we iterate the inversion procedure in 3.2.1, eliminating outlying data points based on their residuals to the reference surface. To determine whether outliers may be present, we calculate the chi-squared misfit between the data and the fit surface based on the data covariance matrix and the residual vector, *r*:

$$\chi^2 = r^T \mathbf{C}^{-1} r \qquad 8$$

To determine whether this misfit statistic indicates consistency between the polynomial surface and the data we use a P statistic, which gives the probability that the given χ^2 value would be obtained from a random Gaussian distribution of data points with a covariance matrix **C**. If the probability is less than 0.025, we perform some further filtering/editing: we calculate the RDE of the scaled residuals, eliminate any pairs containing a segment whose scaled residual magnitude is larger than three times that value, and repeat the remaining segments.

After each iteration, any column of **G** that has a uniform value (i.e. all the values are the same) is eliminated from the calculation, and the corresponding value of the left-hand side of equation 7 is set to zero. Likewise, if the inverse problem has become less than overdetermined (i.e., the number of data is smaller than the number of unknown values they are constraining), the polynomial columns of **G** are eliminated one by one until the number of data is greater than the number of unknowns. Columns are eliminated in descending order of the sum of *x* and *y* degrees, and when there is a tie between columns based on this criterion, the column with the larger *y* degree is eliminated first.

This fitting procedure is continued until no further segments are eliminated. If more than three complete cycles that passed the initial editing steps are eliminated in this way, the surface is assumed to be too complex for a simple polynomial approximation, and we proceed as follows:

(*i*) the fit and its statistics are reported based on the complete set of pairs that passed the initial editing steps (valid pairs), using a planar (x degree = y_degree = 1) fit in *x* and *y*.

(*ii*) the *ref_surf/complex_surface_flag* is set to 1.

The misfit parameters are reported in the *ref_surf* group: The final chi-squared statistic is reported as *ref_surf/misfit_chi2r*, equal to the chi-squared statistic divided by the number of degrees of freedom in the solution; the final RMS of the scaled residuals is reported as *ref_surf/misfit_rms*.

3.3 Reference-shape Correction Error Estimates

We first calculate the errors in the corrected surface heights for segments included in the reference-surface fit. We form a second covariance matrix, $\mathbf{C}_1$, whose diagonal elements are the maximum of the squares of the segment errors and $<r^2>$. We estimate the covariance matrix for the height estimates:

$$\mathbf{C}_m = \mathbf{G}^{-g}\mathbf{C}_1\mathbf{G}^{-gT} \tag{9}$$

The square roots of the diagonal values of $\mathbf{C}_m$ give the estimated errors in the surface-polynomial and height estimates due to short-spatial-scale errors in the segment heights. If there are N_{coeff} coefficients in the surface-shape polynomial, and $N_{shape\text{-}cycles}$ cycles included in the surface-shape fit, then the first N_{coeff} diagonal elements of $\mathbf{C}_m$ give the square of the errors in the surface-shape polynomial and the last $N_{shape\text{-}cycles}$ give the errors in the surface heights for the cycles included in the fit. The portion of $\mathbf{C}_m$ that refers only to the surface shape and surface-shape change components is $\mathbf{C}_{m,s}$.

3.4 Calculating corrected height values for repeats with no selected pairs

Once the surface polynomial has been established from the edited data set, corrected heights are calculated for the unselected cycles (*i.e.* those from which all pairs were removed in the editing steps): For the segments among these cycles, we form a new surface and slope-change design matrix, $[\mathbf{S}, \mathbf{S}_t]$ and multiply it by $[\mathbf{s}, \mathbf{s}_t]$ to give the surface-shape correction:

$$\boldsymbol{z}_c = \boldsymbol{z} - [\mathbf{S}, \mathbf{S}_t][\mathbf{s}, \mathbf{s}_t] \tag{10}$$

Here $\boldsymbol{s}$ is the surface-shape polynomial, and $\mathbf{s}_t$ is the slope-change-rate estimate. This gives up to fourteen corrected-height values per unselected cycle. From among these, we select the segment with the minimum error, as calculated in the next step.

The height errors for segments from cycles not included in the surface-shape fit are calculated:

$$\boldsymbol{\sigma}_{z,c}^2 = diag\left([\mathbf{S}, \mathbf{S}_t]\mathbf{C}_{m,s}[\mathbf{S}, \mathbf{S}_t]^T\right) + \sigma_z^2 \tag{11}$$

Here $\boldsymbol{\sigma}_z$ is the error in the segment height, and $\boldsymbol{\sigma}_{z,c}$ is the error in the corrected height. The results of these calculations give a height and a height error for each unselected segment. To obtain a corrected elevation for each repeat that contains no selected pairs, we identify the segment from that repeat that has the smallest error estimate, and report the value z_c as that repeat's *ptx/h_corr*, and use $\boldsymbol{\sigma}_{z,c}$ as its error (*/ptx/h_corr_sigma)*.

3.5 Calculating systematic error estimates

The errors that have been calculated up to this point are due to errors in fitting segments to photon-counting data and due to inaccuracies in the polynomial fitting model. Additional error components can result from more systematic errors, such as errors in the position of ICESat-2 as derived from POD, and pointing errors from PPD. These are estimated in the ATL06 *sigma_geo_xt*, *sigma_geo_at*, and *sigma_geo_r* parameters, and their average for each repeat is reported in the *cycle_stats* group under the same parameter names. The geolocation component of the total height is the product of the geolocation error and the surface slope, added in quadrature with the vertical height error:

$$\sigma_{h,systematic} = \left[\left(\frac{dh}{dx}\sigma_{geo,AT}\right)^2 + \left(\frac{dh}{dy}\sigma_{geo,XT}\right)^2 + \sigma_{geo,r}^2\right]^{1/2} \tag{12}$$

For selected segments, which generally come from pairs containing two high-quality height estimates, *dh/dy* is estimated from the ATL06 *dh_fit_dy* parameter. For unselected segments, it is based on the *y* component of the reference-surface slope, as calculated in section 4.2. The error for a single segment's corrected height is:

$$\sigma_{h,total} = \left[\sigma_{h,systematic}^2 + \sigma_{h,c}^2\right]^{1/2} \tag{13}$$

This represents the total error in the surface height for a single corrected height. In most cases, error estimates for averages of ice-sheet quantities will depend on errors from many segments from different reference points, and the spatial scale of the different error components will need to be taken into account in error propagation models. To allow users to separate these effects, we report both the uncorrelated error, */ptx/h_corr_sigma*, and the component due only to systematic errors, */ptx/h_corr_sigma_systematic*. The total error is the quadratic sum of the two, as described in equation 13.

3.6 Calculating shape-corrected heights for crossing-track data

Locations where groundtracks cross provide opportunities to check the accuracy of measurements by comparing surface-height estimates between the groundtracks, and also offers the opportunity to generate elevation-change time series that have more temporal detail than the 91-day repeat cycle can offer for repeat-track measurements.
At these crossover points, we use the reference surface calculated in 3.5 to calculate corrected elevations for the crossing tracks. We refer to the track for which we have calculated the reference surface as the *datum* track, and the other track as the *crossing* track. To calculate corrected surface heights for the crossing ICESat-2 orbits, we first select all data from the crossing orbit within a distance *L_search_XT* of the updated reference point on the datum track. For most datum reference points, this will yield no crossing data, in which case the calculation for that datum point terminates. If crossing data are found, we then calculate the coordinates of these points in the reference point's along-track and across-track coordinates. This calculation begins by transforming the crossing-track data into local northing and easting coordinates relative to the datum reference-point location:

$$\delta N_c = \frac{\pi R_e}{180}(lat_c - lat_d) \tag{14}$$

$$\delta E_c = \frac{\pi R_e}{180}(lon_c - lon_d)\cos(lat_c)$$

Here (lat_d, lon_d) are the coordinates of the adjusted datum reference point, (lat_c, lon_c) are the coordinates of the points on the crossing track, and R_e is the local radius of the WGS84 ellipsoid. We then convert the northing and easting coordinates into along-track and across-track coordinates based on the azimuth ϕ of the datum track:

$$x_c = \delta N_c\cos(\phi) + \delta E_c\sin(\phi) \tag{15}$$
$$y_c = \delta N_c\sin(\phi) - \delta E_c\cos(\phi)$$

Using these coordinates, we proceed as we did in 3.4 and 3.5: we generate $\mathbf{S}_k$ and $\mathbf{S}_{kt}$ matrices, use them to correct the data and to identify the data point with the smallest error for each crossing cycle. We report the time, error estimate, and corrected height for the minimum-error datapoint from each cycle, as well as the location, pair, and track number corresponding to the datum point in the */ptx/crossing_track_data* group. Because the crossing angles between the tracks are oblique at high latitudes, a particular crossing track may appear in a few subsequent datum points; in these cases, we expect that the error estimates should vary with the distance between the crossing track and the datum track, so that the point with the minimum error should correspond to the precise crossing location of the two tracks.

To help evaluate the quality of crossing-track data we calculate the *along_track_rss* parameter for each crossing-track measurement. This parameter gives the RSS of the differences between each segment's endpoint heights and the heights of the previous and subsequent segments. A segment that is consistent with the previous and next segments in slope and elevation will have a small value for this parameter, a segment that is inconsistent (and thus potentially in error) will have a large value. Crossing-track measurements that have values greater than 10 m are excluded form ATL11 and do not appear in the dataset.

3.7 Calculating parameter averages

ATL11 contains a variety of parameters that mirror parameters in ATL06, but are averaged to the 140-m ATL11 resolution. Except where noted otherwise, these quantities are weighted averages of the corresponding ATL06 values. For selected pairs (i.e. those included in the reference-surface fit), the parameters are averaged over the selected segments from each cycle, using weights derived from their formal errors, *h_li_sigma*. The parameter weighted average for the N_k segments from cycle k is then:

$$\langle q \rangle = \frac{\sum_{i=1}^{N_k} \left|\sigma_i^{-2}\right| q_i}{\sum_{i=1}^{N_k} \left|\sigma_i^{-2}\right|} \qquad 16$$

Here q_i are the parameter values for the segments. For repeats with no selected pairs, recall that the corrected height for only one segment is reported in */ptx/h_corr*; for these, we simply report the corresponding parameter values for that selected segment.

3.8 Output data editing

The output data product includes cycle height estimates only for those cycles that have non-systematic error estimates (*/ptx/h_corr_sigma*) less than 15 m. All other heights (and their errors) are reported as *invalid*.

4.0 LAND ICE PRODUCTS: LAND ICE H (T)(ATL 11/L3B)

Each ATL11 file contains data for a single reference ground track, for one of the subregions defined for ATLAS granules (see Figure 6-3). The ATL11 consists of three top-level groups, one for each beam pair (*pt1, pt2, pt3*). Within each pair-track group, there are datasets that give the corrected heights for each cycle, their errors, and the reference-point locations. Subgroups (*cycle_stats,* and *ref_surf)* provide a set of data-quality parameters, and ancillary data describing the fitting process, and use the same ordering and coordinates as the top-level group (i.e. any dataset within the */ptx/cycle_stats* and */ptx/ref_surf* groups refers to the same latitude, longitude, and reference points as the corresponding measurements in the */ptx/* groups.) The *crossing_track_data* group gives height measurements at crossover locations, and has its own set of locations and

4.1 File naming convention

ATL11 files are named in the following format:

ATL11_*ttttgg_cccc_rrr_vv*.h5

Here *tttt* is the rgt number, *gg* is the granule-region number, *cccc* gives the first and last cycles of along-track data included in the file (e.g. _0308_ would indicate that cycles three through eight, inclusive, might be included in the along-track solution), and *rrr* is the release number. and *vv* is the version number, which is set to one the first time a granule is generated for a given data release, and is incremented by one if the granule is regenerated.

4.2 */ptx* group

Table 4-1 shows the datasets in the *ptx* groups. This group gives the principal output parameters of the ATL11. The corrected repeat measurements are in */ptx/h_corr*, which gives improved height measurements based on a surface fit to valid data at paired segments. The associated reference coordinates, */ptx/latitude* and */ptx/longitude* give the reference point location, with averaged times per repeat, */ptx/delta_time*. For repeats with no selected pairs, the corrected height is that from the selected segment with the lowest error. Two error metrics are given in */ptx/h_corr_sigma* and */ptx/h_corr_sigma_systematic*. The first gives the error component due to ATL06 range errors and due to uncertainty in the reference surface. The second gives the component due to geolocation and radial-orbit errors that are correlated at scales larger than one reference point; adding these values in quadrature gives the total per-cycle error. Values are only reported for */ptx/h_corr, /ptx/h_corr_sigma,* and */ptx/h_corr_sigma_systematic* for those cycles whose uncorrelated errors are less than 15 m; all others are reported as *invalid.* A */ptx/quality_summary* is included for each cycle, based on fit statistics from ATL06.

Table 4-1 Parameters in the */ptx/* group

Parameter	Units	Dimensions	Description
cycle_number	counts	$1xN_{cycles}$	Cycle number for each column of the data
latitude	degrees North	$N_{pts}\times 1$	Reference point latitude
longitude	degrees East	$N_{pts}\times 1$	Reference point longitude
ref_pt	counts	$N_{pts}\times 1$	The reference point number, *m*, counted from the equator crossing of the RGT.
delta_time	seconds	$N_{pts}\times N_{cycles}$	mean GPS time for the segments for each cycle
h_corr	meters	$N_{pts}\times N_{cycles}$	the mean corrected height
h_corr_sigma	meters	$N_{pts}\times N_{cycles}$	the formal error in the corrected height
h_corr_sigma_systematic	meters	$N_{pts}\times N_{cycles}$	the magnitude of the RSS of all errors that might be correlated at scales larger than a single reference point (e.g. pointing errors, GPS errors, etc)
quality_summary	counts	$N_{pts}\times N_{cycles}$	summary flag: zero indicates high-quality cycles: where min(signal_selection_source)<=1 and min(SNR_significance) < 0.02, and ATL06_summary_zero_count >0.

4.3 */ptx/ref_surf* group

Table 4-2 describes the */ptx/ref_surf* group. This group includes parameters describing the reference surface fit at each reference point. The polynomial coefficients are given in

/ptx/poly_ref_surf, sorted first by total degree, then by x-component degree. Because the polynomial degree is chosen separately for each reference point, enough columns are provided in the */ptx/poly_ref_surf* and */ptx/poly_ref_surf_sigma* to accommodate all possible components up to 2rd degree in y and 3th degree in x, and absent values are filled in with zeros. The correspondence between the columns of the polynomial fields and the exponents of the x and y terms are given in the */ptx/poly_exponent_x* and */ptx/poly_exponent_y* fields. The time origin for the slope change is given in the group attribute */ptx/slope_change_t0.*

Table 4-2 Parameters in the */ptx/ref_surf* group

Parameter	Units	Dimensions	Description
dem_h	Meters	$N_{pts}\times 1$	DEM elevation, derived from the ATL06 *dem_h* parameter
geoid_h	Meters	$N_{pts}\times 1$	geoid elevation, derived from the ATL06 *geoid_h* parameter
complex_surface_flag	counts	$N_{pts}\times 1$	0 indicates that normal fitting was attempted, 1 indicates that the signal selection algorithm rejected too many repeats, and only a linear fit was attempted
rms_slope_fit	counts	$N_{pts}\times 1$	the RMS of the slope of the fit polynomial within 50 m of the reference point
e_slope	counts	$N_{pts}\times 1$	the mean East-component slope for the reference surface within 50 m of the reference point
n_slope	counts	$N_{pts}\times 1$	the mean North-component slope for the reference surface within 50 m of the reference point
at_slope	Counts	$N_{pts}\times 1$	Mean along-track component of the slope of the reference surface within 50 m of the reference point
xt_slope		$N_{pts}\times 1$	Mean across-track component of the slope of the reference surface within 50 m of the reference point
deg_x	counts	$N_{pts}\times 1$	Maximum degree of non-zero polynomial components in x
deg_y	counts	$N_{pts}\times 1$	Maximum degree of non-zero polynomial components in y
poly_exponent_x	counts	*1x8*	Exponents for the x factors in the surface polynomial
poly_exponent_y	counts	*1x8*	Exponents for the y factors in the surface polynomial
poly_coeffs	counts	$N_{pts}\times 8$	polynomial coefficients (up to degree 3), for polynomial components scaled by 100 m

poly_ref_coeffs_sigma	counts	$N_{pts}\times 8$	formal errors for the polynomial coefficients
ref_pt_number	counts	$N_{pts}\times 1$	Ref point number, counted from the equator crossing along the RGT.
x_atc	meters	$N_{pts}\times 1$	Along-track coordinate of the reference point, measured along the RGT from its first equator crossing.
y_atc	meters	$N_{pts}\times 1$	Across-track coordinate of the reference point, measured along the RGT from its first equator crossing.
rgt_azimuth	degrees	$N_{pts}\times 1$	Reference track azimuth, in degrees east of local north
slope_change_rate_x	$years^{-1}$	$N_{pts}\times 1$	rate of change of the x component of the surface slope
slope_change_rate_y	$years^{-1}$	$N_{pts}\times 1$	rate of change of the y component of the surface slope
slope_change_rate_x_sigma	$years^{-1}$	$N_{pts}\times 1$	Formal error in the rate of change of the x component of the surface slope
slope_change_rate_y_sigma	$years^{-1}$	$N_{pts}\times 1$	Formal error in the rate of change of the y component of the surface slope
misfit_chi2r	meters	$N_{pts}\times 1$	misfit chi square, divided by the number of degrees in the solution
misfit_rms	meters	$N_{pts}\times 1$	RMS misfit for the surface-polynomial fit
fit_quality	counts	$N_{pts}\times 1$	Indicates quality of the fit: 0: no problem identified 1: One or more polynomial coefficient errors larger than 10 2: One or more components of the surface slope has magnitude larger than 0.2 3: Conditions 1 and 2 both true.

The slope of the fit surface is given in the *ref_surf/n_slope* and *ref_surf/e_slope* parameters in the local north and east directions; the corresponding slopes in the along-track and across-track directions are given in the *ref_surf/xt_slope* and *ref_surf/yt_slope* parameters. For the along-track points, the surface slope is calculated by evaluating the correction-surface polynomial for a 10-m spaced grid of points extending ±50 m in x and y around the reference point, and calculating the mean slopes of these points. The calculation is performed in along-track coordinates and then projected onto the local north and east vectors. The *rms_slope_fit* is derived from the same set of points, and is calculated as the RMS of the standard deviations of the slopes calculated from adjacent grid points, in *x* and *y*.

4.4 /ptx/cycle_stats group

The */ptx/cycle_stats* group gives summary information about the segments present for each reference point. Most parameters are averaged according to equation 14, but for others (e.g. */ptx/signal_selection_flag_best*, which is the minimum of the signal selection flags for the cycle) **Table 4-3** describes how the summary statistics are derived.

Table 4-3 Parameters in the */ptx/cycle_stats* group

Parameter	Units	Dimensions	Description
ATL06_summary_zero_count	counts	$N_{pts} \times N_{cycles}$	Number of segments with *atl06_quality_summary*=0 (0 indicates the best-quality data)
h_rms_misfit	meters	$N_{pts} \times N_{cycles}$	Weighted-average RMS misfit between PE heights and along-track land-ice segment fit
r_eff	counts	$N_{pts} \times N_{cycles}$	Weighted-average effective, uncorrected reflectance for each cycle.
tide_ocean	meters	$N_{pts} \times N_{cycles}$	Weighted-average ocean tide for each cycle
dac	meters	$N_{pts} \times N_{cycles}$	Dynamic atmosphere correction (mainly the effect of atmospheric pressure on floating-ice elevation).
cloud_flg_atm	counts	$N_{pts} \times N_{cycles}$	Minimum cloud flag from ATL06: Flag indicates confidence that clouds with OT* > 0.2 are present in the lower 3 km of the atmosphere based on ATL09
cloud_flg_asr	counts	$N_{pts} \times N_{cycles}$	Minimum apparent-surface-reflectance -based cloud flag from ATL06: Flag indicates confidence that clouds with OT > 0.2 are present in the lower 3 km of the atmosphere based on ATL09
bsnow_h	meters	$N_{pts} \times N_{cycles}$	Weighted-average blowing snow layer height for each cycle
bsnow_conf	counts	$N_{pts} \times N_{cycles}$	Maximum bsnow_conf flag from ATL06: indicates the greatest (among

Parameter	Units	Dimensions	Description
			segments) confidence flag for presence of blowing snow for each cycle
x_atc	meters	$N_{pts} \times N_{cycles}$	weighted average of pair-center RGT y coordinates for each cycle
y_atc	meters	$N_{pts} \times N_{cycles}$	weighted mean of pair-center RGT y coordinates for each cycle
ref_pt		$N_{pts} \times N_{cycles}$	Ref point number, counted from the equator crossing along the RGT.
seg_count	counts	$N_{pts} \times N_{cycles}$	Number of segments marked as valid for each cycle. Equal to 0 for those cycles not included in the reference-surface shape fit.
min_signal_selection_source	counts	$N_{pts} \times N_{cycles}$	Minimum of the ATL06 signal_selection_source value (indicates the highest-quality segment in the cycle)
min_snr_significance	counts	$N_{pts} \times N_{cycles}$	Minimum of SNR_significance (indicates the quality of the best segment in the cycle)
sigma_geo_h	meters	$N_{pts} \times N_{cycles}$	Root-mean-weighted-square-average total vertical geolocation error due to PPD and POD
sigma_geo_at	meters	$N_{pts} \times N_{cycles}$	Root-mean-weighted-square-average local-coordinate x horizontal geolocation error for each cycle due to PPD and POD
sigma_geo_xt	meters	$N_{pts} \times N_{cycles}$	Root-mean-weighted-square-average local-coordinate y horizontal geolocation error for each cycle due to PPD and POD
h_mean	meters	$N_{pts} \times N_{cycles}$	Weighted-average of surface heights, not including the correction for the reference surface

*OT (optical thickness) is a measure of signal attenuation used in atmospheric calculations. This parameter discussed in ICESat-2 atmospheric products (ATL09)

4.5 */ptx/crossing_track_data* group

The */ptx/crossing_track_data* group (Table 4-4) contains elevation data at crossover locations. These are locations where two ICESat-2 pair tracks cross, so data are available from both the datum track, for which the granule was generated, and from the crossing track. The data in this group represent the elevations and times from the crossing tracks, corrected using the reference surface from the datum track. Each set of values gives the data from a single segment on the crossing track, that was selected as having the minimum error among all segments on the crossing track within the 2 *L_search_XT* –by-2 *L_search_AT* window around the reference point on the datum track. The systematic errors are evaluated based on the magnitude of the reference-surface slope and the magnitude of the horizontal geolocation error of the crossing-track data. Attributes for the group specify the track number and pair-track number of the crossing track.

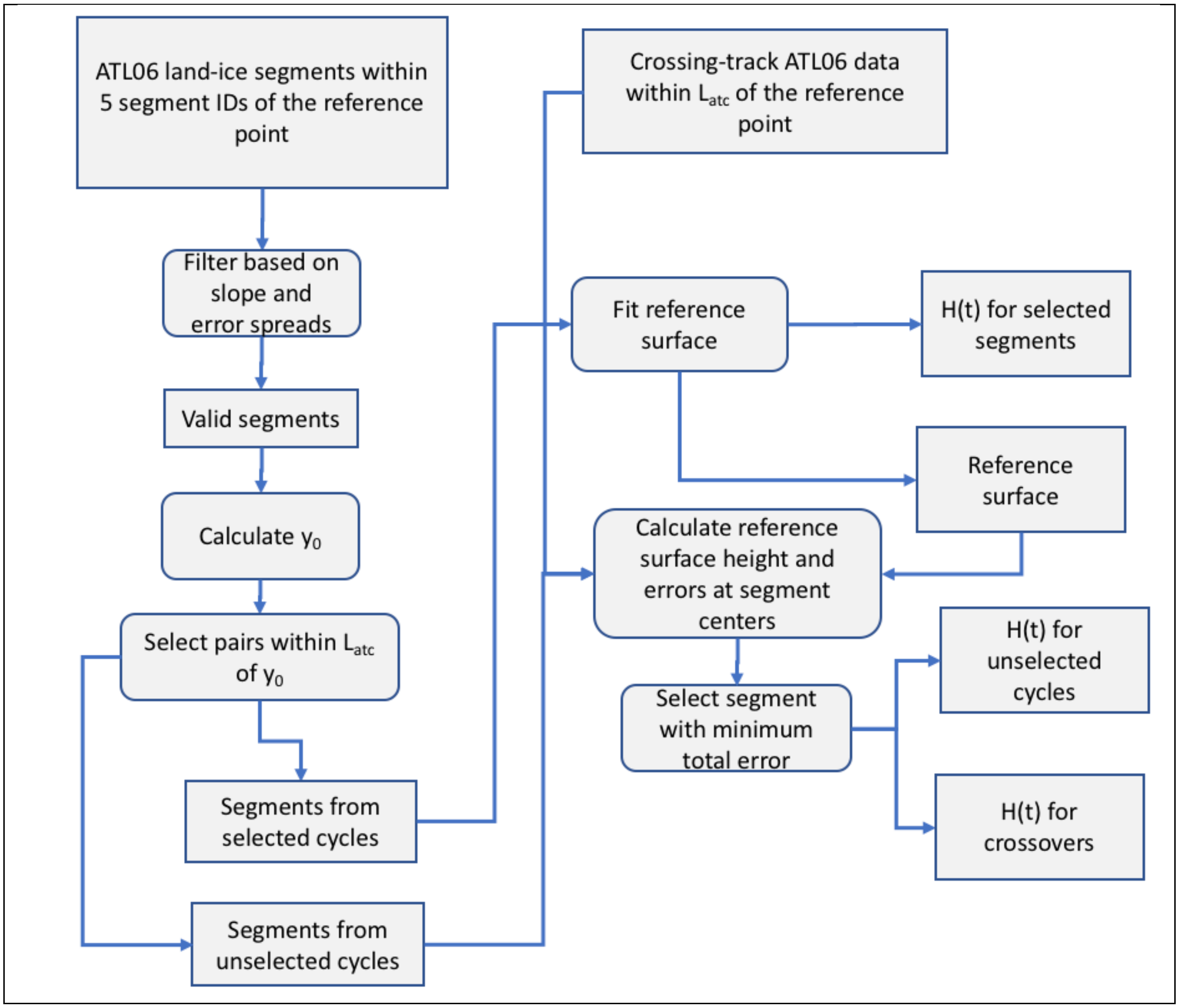
Table 4-4 Parameters in the */ptx/crossing_track_data* group

Parameter	Units	Dimensions	Description
ref_pt	counts	$N_{XO} \times 1$	the reference-point number for the datum track
delta_time	years	$N_{XO} \times 1$	time relative to the ICESat-2 reference epoch
h_corr	meters	$N_{XO} \times 1$	WGS-84 height, corrected for the ATL11 surface shape
h_corr_sigma	meters	$N_{XO} \times 1$	error in the height estimate
h_corr_sigma_systematic	meters	$N_{XO} \times 1$	systematic error in the height estimate
ocean_tide	Meters	$N_{XO} \times 1$	Ocean-tide estimate for the crossing track
dac	Meters	$N_{XO} \times 1$	Dynamic atmosphere correction for the crossing track
latitude	degrees	$N_{XO} \times 1$	latitude of the crossover point
longitude	degrees	$N_{XO} \times 1$	longitude of the crossover point
cycle_number	counts	$N_{XO} \times 1$	Cycle number for the crossing data
rgt	counts	$N_{XO} \times 1$	The RGT number for the crossing data
spot_crossing	counts	$N_{XO} \times 1$	The spot number for the crossing data

atl06_quality_summary	counts	$N_{XO} \times 1$	quality flag for the crossing data derived from ATL06. 0 indicates no problems detected, 1 indicates potential problems
along_track_rss	meters	$N_{XO} \times 1$	Root sum of the squared differences between the heights of the endpoints for the crossing-track segment and the centers of the previous and next segments

5.0 ALGORITHM IMPLEMENTATION

Figure 5-1 Flow Chart for ATL11 Surface-shape Corrections



The following steps are performed for each along-track reference point.

1. Segments with *segment_id* within *N_search/2* of the reference-point number, are selected.
2. Valid segments are identified based on estimated errors, the *ATL06_quality_summary* parameter, and the along- and across-track segment slopes. Valid pairs, containing valid measurements from two different beams, are also identified.
3. The location of the reference point is adjusted to allow the maximum number of repeats with at least one valid pair to fall within the across-track search distance of the reference point.

4. The reference surface is fit to pairs with two valid measurements within the search distance of the reference point. This calculation also produces corrected heights for the selected pairs and the errors in the correction polynomial coefficients.
5. The correction surface is used to derive corrected heights for segments not selected in steps 1-3, and the height for the segment with the smallest error is selected for each
6. The reference surface is used to calculate heights for external (pre-ICESat-2) laser altimetry data sets and crossover ICESat-2 data.

A schematic of this calculation is shown in Figure 5-1.

5.1.1 Select ATL06 data for the current reference point

Inputs:
ref_pt: segment number for the current refererence point
track_num: The track number for current point
pair_num: The pair number for the current point
Outputs:
D_ATL06: ATL06 data structure
Parameters:
N_search: number of segments to search, around ref_pt, equal to 5.
Algorithm:
1. For each along-track point, load all ATL06 data from track *track_num* and pair *pair_num* that have *segment_id* within *N_search* of *ref_pt:* These segments have *ref_pt - N_search <=segment_id <= ref_pt +N_search.*
2. Reject any data that have *y_atc* values more than 500 m distant from the nominal pair-track centers (3200 m for pair 1, 0 m for pair 2, -3200 m for pair 3).

5.1.2 Select pairs for the reference-surface calculation

Inputs:
ref_pt: reference point number for the current fit
x_atc_ctr: Along-track coordinate of the reference point
D_ATL06: ATL06 data structure
pair_data: Structure describing ATL06 pairs, includes mean of strong/weak beam *y_atc* and *dh_fit_dy*
Outputs:
validity flags for each segment:
valid_segs.x_slope: Segments identified as valid based on x-slope consistency
valid_segs.data: Segments identified as valid based on ATL06 parameter values.
Validity flags for each pair:
valid_pairs: Pairs selected for the reference-surface calculation
valid_pairs.y_slope: Pairs identified as valid based on y-slope consistency
y_polyfit_ctr: y center of the slope regression
ref_surf/complex_surface_flag: Flag indicating 0: non-complex surface, 1: complex surface.

Parameters:

L_search_XT: The across-track search distance.

N_search: Along-track segment search distance

seg_sigma_threshold_min: Minimum threshold for accepting errors in segment heights, equal to 0.05 m.

Algorithm:

1. Flag valid segments based on ATL06 values.

1a. Count the cycles that contain at least one pair that has *atl06_quality_flag*=0 for both segments. If this number is greater than *N_cycles/3,* set *ref_surf/complex_surface_flag*=0 and set *valid_segs.data* to 1 for segments with *ATL06_quality_summary* equal to 0. Otherwise, set *ref_surf/complex_surface_flag*=1 and set *valid_segs.data* to 1 for segments with *snr_significance < 0.02*.

1b. Define *seg_sigma_threshold* as the maximum of 0.05 or three times the median of *sigma_h_li* for segments with *valid_segs.data* equal to 1. Set *valid_segs.data* to 1 for segments with *h_sigma_li* less than this threshold and *ATL06_quality_summary* equal to 0.

1c. Define *valid_pairs.data:* For each pair of segments, set *valid_pairs.data* to 1 when both segments are marked as valid in *valid_segs.data*.

2. Calculate representative values for the *x* and *y* coordinate for each pair, and filter by distance.

2a. For each pair containing two defined values, set *pair_data.x* to the segments' *x_atc* value, and *pair_data.y* to the mean of the segments' *y_atc* values.

2b. Calculate *y_polyfit_ctr*, equal to the median of *pair_data.y* for pairs marked valid in *valid_pairs.data.*

2c. Set *valid_pairs.ysearch* to 1 for pairs with |*pair_data.y - y_polyfit_ctr*| < *L_search_XT*.

3. Select pairs based on across-track slope consistency

3a. Define *pairs_valid_for_y_fit*, for the across-track slope regression if they are marked as valid in *valid_pairs.data*, and *valid_pairs.ysearch*, not otherwise.

3b. Choose the degree of the regression for across-track slope

-If the valid pairs contain at least two different *x_atc* values (separated by at least 18 m), set the along-track degree, *my_regression_y_degree*, to 1, 0 otherwise.

-If valid pairs contain at least two different *ref_surf/y_atc* values (separated by at least 18 m), set the across-track degree, *my_regression_y_degree*, to 1, 0 otherwise.

3c. Calculate the formal error in the y slope estimates: *y_slope_sigma* is the RSS of the *h_li_sigma* values for the two beams in the pair divided by the difference in their *y_atc* values. Based on these, calculate *my_regression_tol*, equal to the maximum of 0.01 or three times the median of *y_slope_sigma* for valid pairs (*pairs_valid_for_y_fit)*.

3d. Calculate the regression of *dh_fit_dy* against *pair_data.x* and *pair_data.y* for valid pairs (*pairs_valid_for_y_fit*). The result is *y_slope_model*, which gives the variation of *dh_fit_dy* as a function of *x_atc* and *y_atc*. Calculate *y_slope_resid*, the residuals between the *dh_fit_dy* values and *y_slope_model* for all pairs in *pair_data*.

3e. Calculate *y_slope_threshold*, equal to the maximum of *my_regresion_tol* and three times the RDE of *y_slope_resid* for valid pairs.

3f. Mark all pairs with |*y_slope_resid*| > y*_slope_threshold* as invalid. Re-establish *pairs_valid_for_y_fit (*based on *valid_pairs.data*, *valid_pairs.y_slope* and *valid_pairs.ysearch)*. Return to step 3d (allow two iterations total).

3g. After the second repetition of 3d-f, use the model to mark all pairs with |*y_slope_resid*| less than y*_slope_threshold* with 1 in *valid_pairs.y_slope*, 0 otherwise.

4. Select segments based on along-track slope consistency for both segments in the pair

4a. Define *pairs_valid_for_x_fit*, valid segments for the along-track slope regression*:* segments are valid if they come from pairs marked as valid in *valid_pairs.data* and *valid_pairs.ysearch*, not otherwise.

4b. Choose the degree of the regression for along-track slope

-If valid segments contain at least two different *x_atc* values set the along-track degree, *mx_regression_x_degree*, to 1, 0 otherwise.

-If valid segments contain at least two different *y_atc* values, set the across-track degree, *mx_regression_y_degree*, to 1, 0 otherwise.

4c. Calculate along-track slope regression tolerance, *mx_regression_tol*, equal to the maximum of either 0.01 or three times the median of the *dh_fit_dx_sigma* values for the valid pairs.

4d. Calculate the regression of *dh_fit_dx* against *pair_data.x* and *pair_data.y* for valid segments (*pairs_valid_for_x_fit*). The result is *x_slope_model*, which gives the variation of *dh_fit_dx* as a function of *pair_data.x* and *pair_data.y*. Calculate *x_slope_resid*, the residuals between the *dh_fit_dx* and *x_slope_resid* for all segments for this reference point, *seg_x_center* and *y_polyfit_ctr*.

4e. Calculate *x_slope_threshold*, equal to the maximum of either *mx_regression_tol* or three times the RDE of *x_slope_resid* for valid segments.

4f. Mark *valid_segs.x_slope* with |*x_slope_resid*| > *x_slope_threshold* as invalid. Re-establish *valid_pairs.x_slope* when both *valid_segs.x_slope* equal 1. Re-establish *pairs_valid_for_x_fit*. Return to step 4d (allow two iterations total).

4g. After the second repetition of 4d-f, mark all segments with |*x_slope_resid*| less than x*_slope_threshold* with 1 in *seg_valid_xslope*, 0 otherwise. Define *valid_pairs.x_slope* as 1 for pairs that contain two segments with *valid_segs.x_slope=1*, 0 otherwise.

5. Re-establish *valid_pairs.all.* Set equal to 1 if *valid_pairs.x_slope, valid_pairs.y_slope,* and *valid_pairs.data* are all valid.

5a. Identify *unselected_cycle_segs*, as those *D6.cycles* where *valid_pairs.all* are False.

5.1.3 Adjust the reference-point y locaction to include the maximum number of cycles

Inputs:

D_ATL06: ATL06 structure for the current reference point.

valid_pairs: Pairs selected based on parameter values and along- and across-track slopes.

Outputs:

ref_surf/y_atc: Adjusted fit-point center *y.*

valid_pairs: validity masks for pairs, updated to include those identified as valid based on the spatial search around *y_atc_ctr*.

Parameters:

L_search_XT: Across-track search length (equal to 110 m)

Algorithm:

1. Define *y0* as the median of the unique integer values of the pair center y_atc for all valid pairs. Set a range of y values, *y0_shifts*, as round(*y0)* +/- 100 meters in 2-meter increments.

2. For each value of *y0_shifts* (*y0_shift*), set a counter, *selected_seg_cycle_count*, to the number of distinct cycles for which both segments of the pair are contained entirely within the *y* interval [*y0_shift- L_search_XT*, *y0_shift+ L_search_XT*]. Add to this, the number of distinct cycles represented by unpaired segments contained within that interval, weighted by 0.01. The sum is called *score*.

3. Search for an optimal y-center value (with the most distinct cycles). Set *y_best* to the value of *y0_shift* that maximizes *score*. If there are multiple *y0_shift* values with the same, maximum *score*, set to the median of the *y0_shift* values with the maximum *score*.

4. Update *valid_pairs* to include all pairs with *y_atc* within +/- L_search_XT from *y_atc_ctr*.

5.1.4 Calculate the reference surface and corrected heights for selected pairs

Inputs:

D_ATL06: ATL06 structure for the current reference point, containing parameters for each segment:

- *x_atc:* along-track coordinate
- *y_atc*: across-track coordinate
- *delta_t:* time for the segment

pair_data: Structure containing information about ATL06 pairs. Must include:

- *y_atc:* Pair-center across-track coordinates

valid_pairs: Pairs selected based on parameter values and along- and across-track slope.

x_atc_ctr: The reference point along-track x coordinate (equal to *ref_surf/x_atc*).

y_atc_ctr: The reference point along-track x coordinate (equal to *ref_surf/y_atc*)

Outputs:

ref_surf/deg_x: Degree of the reference-surface polynomial in the along-track direction

ref_surf/deg_y: Degree of the reference-surface polynomial in the across-track direction

ref_surf/poly_coeffs: Polynomial coefficients of the reference-surface fit

ref_surf/poly_coeffs_sigma: Formal error in polynomial coefficients of the reference-surface fit

ref_surf/slope_change_rate_x: Rate of change of the x component of the surface slope

ref_surf/slope_change_rate_x_sigma: Formal error in the rate of change of the x component of the surface slope

ref_surf/slope_change_rate_y: Rate of change of the y component of the surface slope

ref_surf/slope_change_rate_y_sigma: Formal error in the rate of change of the y component of the surface slope

r_seg: Segment residuals from the reference-surface model

/ptx/h_corr: Partially filled-in per-cycle corrected height for cycles used in reference surface

/ptx/h_corr_sigma: Partially filled-in per-cycle formal error in corrected height for cycles used in reference surface

ref_surf_cycles: A list of cycles used in defining the reference surface

C_m_surf: Covariance matrix for the reference-polynomial and surface-change model

fit_columns_surf: Mask identifying which components of the combined reference-polynomial and surface-change model were included in the fit.

poly_exponent_x: The x degrees corresponding to the columns of matrix used in fitting the reference surface to the data

poly_exponent_y: The y degrees corresponding to the columns of matrix used in fitting the reference surface to the data

selected_segments: A set of flags indicating which segments were selected by the iterative fitting process.

Partially filled-n per-cycle ATL11 output variables (see table 4-3) for cycles used in reference surface

Parameters:

poly_max_degree_AT: Maximum polynomial degree for the along-track fit, equal to 3.

poly_max_degree_XT: Maximum polynomial degree for the across-track fit, equal to 2.

slope_change_t0: Half the duration of the mission (equal to the time of the last-possible elevation value minus the time of the start of data collection, divided by two).

max_fit_iterations: Maximum number of iterations for surface fitting, with acceptable residuals, equal to 20.

xy_scale: The horizontal scaling value used in polynomial fits, equal to 100 m

t_scale: The time scale used in polynomial fits, equal to seconds in 1 year.

Algorithm:

1. Build the cycle design matrix: **G_zp** is a matrix that has one column for each distinct cycle in *selected_pairs* and one row for each segment whose pair is in *selected pairs*. For each segment, the corresponding row of **G_zp** is 1 for the column matching the cycle for that segment and zero otherwise.

2. Select the polynomial degree.

The degree of the *x* polynomial, *ref_surf/deg_x,* is: *min*(*poly_max_degree_AT,* maximum(number of distinct values of *round((x_atc- x_atc_ctr)/20)* among the selected segments in any one cycle) -1), and the degree of the *y* polynomial, *ref_surf/deg_y,* is : *min*(*poly_max_degree_XT,* number of distinct values of *round((pair_data.y_atc-y_atc_ctr)/20)* among the selected pairs)

3. Perform an iterative fit for the reference-surface polynomial.

3a. Define *degree_list_x* and *degree_list _y*: This array defines the *x* and *y* degree of the polynomial coefficients in the polynomial surface model. There is one component for each unique degree combination of *x* degrees between 0 and *ref_surf/deg_x* and for *y* degree between 0 and *ref_surf/deg_y* such that *x_degree + y_degree* <= *max(ref_surf/deg_x, ref_surf/deg_y),* except that there is no *x_degree=0* and *y_degree=0* combination. They are sorted first by the sum of the *x* and *y* degrees, then by *x* degree, then by *y* degree.

3b. Define the polynomial fit matrix. **S_fit_poly** has one column for each element of the polynomial degree arrays, with values equal to ((*x_atc* -*x_atc_ctr*)/*xy_scale*)$^{x\_degree}$ ((*y_atc*-*y_atc_ctr*)/*xy_scale*)$^{y\_degree}$. There is one row in the matrix for every segment marked as *selected*.

3c. If the time span is longer than 1.5 years, define slope-change matrices, **S_fit_slope_change**. The first column of the matrix gives the rate of slope change in the x component, equal to *(x_atc- x_atc_ctr)/xy_scale**(*delta_time-slope_change_t0*)/*t_scale*. The second column gives the rate of slope change in the y component, equal to (*y_atc-y_atc_ctr*)/*xy_scale**(*delta_time-slope_change_t0*)/*t_scale*.

3d. Build the surface matrix, **G_surf**, and the combined surface and cycle-height matrix, **G_surf_zp**: The surface matrix is equal to the horizontal catenation of **S_fit_poly**, and, if defined, **S_fit_slope_change.** The combined surface and cycle-height matrix, **G_surf_zp,** is equal to the horizontal catenation of **G_surf** and **G_zp.**

3e. Subset the fitting matrix. Subset **G_surf_zp** by row to include only rows corresponding to selected segments to produce **G** (on the first iteration, all are *selected*). Next, subset **G** by column, first to eliminate all-zero columns, and second to include only columns that are linearly independent from one another: calculate the normalized correlation between each pair of columns in G, and if the correlation is equal to unity, eliminate the column with the higher weighted degree (*poly_wt_sum = x_degree* + 1.1**y_degree,* with the factor of 1.1 chosen to avoid ties). Identify the selected columns in the matrix as *fit_columns*. If more than three of the original surface-change columns have been eliminated, set the *ref_surf/complex_surface_flag* to *True*, mark all columns corresponding to polynomial coefficients of combined x and y degree greater than 1 as *False* in *fit_columns*.

3f. Check whether the inverse problem is under- or even-determined: If the number of *selected_segments* is less than the number of columns of **G**, eliminate remaining columns of **G** in descending order of *poly_wt_sum* until the number of columns of **G** is less than the number of *selected_segments.*

3g. Generate the data-covariance matrix, **C_d**. The data-covariance matrix is a square matrix whose diagonal elements are the squares of the *h_li_sigma* values for the selected segments.

3h. Calculate the polynomial fit. Initialize ***m_surf_zp,*** the reference model, to a vector of zero values, with one value for each column of **G_surf_zp**. Calculate the generalized inverse (equation 7) , of **G**, **G_g.** If the inversion calculation returns an error, or if any row of **G_g** is all-zero (indicating some parameters are not linearly independent), report fit failure and return. Otherwise, multiply **G_g** by the subset of *h_li* corresponding to the selected segment to give ***m,*** containing values for the parameters selected in *fit_columns*. Fill in the components of ***m_surf_zp*** flagged in *fit_columns* with the values in ***m.***

3i. Calculate model residuals for all segments, *r_seg*, equal to *h_li*-***G_surf_dz*** * ***m_surf_zp***. The subset of *r_seg* corresponding to *selected* segments is *r_fit.*

3j. Calculate the fitting tolerance, *r_tol*, equal to three times the RDE of the *r_fit/h_li_sigma* for all *selected* segments. Calculate the reduced chi-squared value for these residuals, *ref_surf/misfit_chi2*, equal to $\boldsymbol{r\_fit}^{T}\mathbf{C\_d}^{-1}\boldsymbol{r\_fit}$. Calculate the *P* value for the misfit, equal to one minus the CDF of a chi-squared distribution with *m-n* degrees of freedom for *ref_surf/misfit_chi2*, where *m* is the number of rows in **G**, and *n* is the number of columns.

3k. If the *P* value is less than 0.025 and fewer than *max_fit_iterations* have taken place, mark all segments for which |*r_seg/h_li_sigma*| < *r_tol* as *selected*, and return to 3e. Otherwise, continue to 3k.

3l. Propagate the errors. Based on the most recent value of **C_d**, generate a revised data-covariance matrix, **C_dp**, whose diagonals values are the maximum of *h_li_sigma*^{2} and RDE(*r_fit*)^2. Calculate the model covariance matrix, **C_m** using equation 9. If any of the diagonal elements of **C_m** are larger than 10^4, report a fit failure and return. Fill in elements of ***m_surf_zp*** that are marked as valid in *fit_columns* with the square roots of the corresponding diagonal elements of **C_m**. If any of the errors in the polynomial coefficients are larger than 10, set *ref_surf/fit_quality*=1.

4. Return a list of cycles used in determining the reference surface in *ref_surf_cycles*. These cycles have columns in **G** that contain a valid pair, and for which the steps 3e and 3j did not eliminate the degree of freedom. For these cycles, partially fill in the values of */ptx/h_corr* and */ptx/h_corr_sigma*, from ***m*** and ***m_sigma***. Similarly, fill in values for */ptx/h_corr_sigma_systematic (*Equation 12*)* and */ptx/delta_time*, as well as all variables in Table 4-3. Set */ptx/h_corr*, */ptx/h_corr_sigma*, */ptx/h_corr_sigma_systematic* to *NaN* for those cycles that have uncorrelated error estimates greater than 15 m.

Values from Table 4-2 defining the fitted reference surface are also reported including *ref_surf/poly_coeffs*, and *ref_surf/poly_coeffs_sigma*, *ref_surf/slope_change_rate_x*, *ref_surf/slope_change_rate_y*, *ref_surf/slope_change_rate_x_sigma*, and *ref_surf/slope_change_rate_y_sigma*.

Return **C_m_surf**, the portion of **C_m** corresponding to the polynomial and slope-change components of **C_m**. Return *selected_cols_surf*, the subset of *selected_cols* corresponding to the surface polynomial and slope-change parameters.

Return the reduced chi-square value for the last iteration, *ref_surf/misfit_chi2r*, equal to *ref_surf/misfit_chi2/(m-n)*.

5.1.5 Calculate corrected heights for cycles with no selected pairs.

Inputs:

C_m_surf: Covariance matrix for the reference-surface model.
degree_list_x, *degree_list_y:* List of x-, y-, degrees for which the reference-surface calculation attempted an estimate.
selected_cols_surf: Parameters of the combined reference-surface and slope-change model for which the inversion returned a value. There should be one value for each row/column of **C_m_surf**.
x_atc_ctr, y_atc_ctr: Center point for the surface fit (equal to *ref_surf/x_atc, ref_surf/y_atc*)
selected_segments: Boolean array indicating segments selected for the reference-surface calculation
valid_segs.x_slope: Segments identified as valid based on x-slope consistency
valid_segs.data: Segments identified as valid based on ATL06 parameter values.
pair_number: Pair number for each segment
h_li: Land-ice height for each segment
h_li_sigma: Formal error in *h_li*.
/ptx/h_corr: Partially filled-in per-cycle corrected height
/ptx/h_corr_sigma: Partially filled-in per-cycle corrected height error

ref_surf/poly_coeffs: Polynomial coefficients from 2-d reference-surface fit
ref_surf_cycles: A list of cycles used in defining the reference surface
ref_surf/slope_change_rate_x, ref_surf/slope_change_rate_y: Rate of change of the x and y components of the surface slope
ref_surf/N_slope, ref_surf/E_slope: slope components of reference surface
sigma_geo_r: Radial component of the geolocation error for the crossing track
D_ATL06: ATL06 data structure
Partially filled-in per-cycle ATL11 output variables (see table 4-3)

Outputs:
/ptx/h_corr: Per-cycle corrected height
/ptx/h_corr_sigma: Per-cycle corrected height error
selected_segments: A set of arrays listing the selected segments for each cycle.
Per-cycle ATL11 output variables (see table 4-3).

Algorithm:
1. Identify the segments marked as valid in *valid_segs.data* and *valid_segs.x_slope* that are not members of the cycles in *ref_surf_cycles*. Label these as *non_ref_segments.*
2. Build **G_other**, a polynomial-fitting matrix for the *non_ref_segments.* **G_other** will include only the polynomial components listed in *degree_list_x* and *degree_list_y*, and (if the mission has been going on for at least 1.5 years) the slope-change components. Multiply **G_other** by [*ref_surf/poly_coeffs*, *ref_surf/slope_change_rate_x, ref_surf/slope_change_rate_y*] to give corrected heights, *z_kc*.
3. Take the subset of **G_other** corresponding to the components in *fit_cols_surf* to make **G_other_surf**. Propagate the polynomial surface errors and surface-height errors for *non_ref_segments* based on **G_other_surf**, **C_m_surf,** and *h_li_sigma* using equation 11. These errors are *z_kc_sigma*.
4. Identify the segments in *non_ref_segments* for each cycle, and, from among these, select the one with the smallest *z_kc_sigma.* If, for this cycle, *z_kc_sigma* is less than 15 m, fill in the corresponding values of */ptx/h_corr* and */ptx/h_corr_sigma*. For cycles containing no valid segments, report invalid data as NaN. Similarly, fill in the variables in Table 4-3, with the value from the segment with the smallest *z_kc_sigma*.

5.1.6 Calculate corrected heights for crossover data points

Inputs:
C_m_surf: Covariance matrix for the reference surface model.
C_m_surf: Covariance matrix for the reference-surface model.
x_atc_ctr, y_atc_ctr: Center point for the surface fit, in along-track coordinates
lat_d, lon_d: Latitude and longitude for the adjusted datum reference point (from */ptx/latitude, /ptx/longitude*)
PT: Pair track for the surface fit
RGT: RGT for the surface fit
ref_surf/rgt_azimuth: The azimuth of the RGT, relative to local north
lat_c, lon_c: Location for crossover data
time_c: Time for crossover data

h_c: Elevations for crossover data
sigma_h_c: Estimated errors for crossover data
Outputs:
ref_pt: reference point (for the reference track)
pt: pair track for the crossing-track points
crossing_track_data/rgt: Reference ground track for the crossing-track point
crossing_track_data/delta_time: time for the crossing-track point
crossing_track_data/h_corr: corrected elevation for the crossing-track points
crossing_track_data/h_corr_sigma: error in the corrected elevation for the crossing_track points
crossing_track_data/h_corr_sigma_systematic: Error component in the corrected elevation due to pointing and orbital errors.
crossing_track_data/along_track_rss:
Parameters:
L_search_XT: Across-track search distance
Algorithm (executed independently for the data from each cycle of the mission):

1. Project data points into the along-track coordinate system:
   - 1a: Calculate along-track and across-track vectors:
     - x_hat=[cos(ref_surf/rgt_azimuth), sin(ref_surf/rgt_azimuth)]
     - y_hat=[sin(ref_surf/rgt_azimuth), -cos(ref_surf/rgt_azimuth)]
   - 1b. Calculate the R_earth, the WGS84 radius at lat_d.
   - 1c: Project the crossover data points into a local projection centered on the fit center:
     - N_d= R_earth (lat_c-lat_d)
     - E_d= R_earth cos(lat_d) (lon_c-lon_d)
   - 1d: Calculate the x and y coordinates for the data points, relative to the fit-center point:
     - dx_c=<x_hat, [E_c, N_c]>
     - dy_c=<y_hat, [E_c, N_c]>

   Here <**a,b**> is the inner (dot) product of **a** and **b**.
2. Calculate the fitting matrix using equation 6.
3. Calculate the errors at each point using the fitting matrix and *C_m*, using on equation 11.
4. Select the minimum-error data point and report the values in Table 4-1.
5. Calculate the systematic error in the corrected height:
   crossing_track_data/h_sigma_sigma_systematic = (*sigma_geo_r*^{2}+ (*N_d ref_surf/n_slope*)^2 + ((*E_d ref_surf/e_slope*)^2)$^{1/2}$
6. Calculate the along-track RSS for the selected segment. For each selected crossing segment calculate the endpoint heights (equal to the segment center height plus or minus 20 meters times the segment's along-track slope), and calculate the RSS of the differences between these heights and the center heights of the previous and subsequent segments. If this RSS difference is greater than 10 m for any cycle, do not report any parameters for that segment's cycle.

5.1.7 Provide error-averaged values for selected ATL06 parameters

Inputs:
ATL06 data structure: ATL06 data to be averaged

Selected_segments: A set of arrays listing the selected segments for each cycle.
Paramteter_list: A list of parameters to be averaged

Outputs:

Parameter_averages: One value for each parameter and each cycle

Algorithm:

1. For each cycle, select the values of *h_li_sigma* based on the values within *selected_segments*. Calculate a set of weights, *w_i*, such that the sum of the weights is equal to 1 and each weight is proportional to the inverse square of *h_li_sigma*. If only one value is present in *selected_segments*, *w_1*=1.
2. For each parameter, multiply the weights for each cycle by the parameter values, report the averaged value in *parameter_averages*.

5.1.8 Provide miscellaneous ATL06 parameters

Inputs:

ATL06 data structure: ATL06 data to be averaged
Selected_segments: A set of arrays listing the selected segments for each cycle.

Outputs:

Weighted-averaged parameter values, with one value per cycle, filled in with NaN for cycles with no selected segments

cycle_stats/h_robust_sprd
cycle_stats/h_li_rms_mean
cycle_stats/r_eff
cycle_stats/tide_ocean
cycle_stats/dac
cycle_stats/bsnow_h
cycle_stats/x_atc
cycle_stats/y_atc
cycle_stats/sigma_geo_h
cycle_stats/sigma_geo_at
cycle_stats/sigma_geo_xt
cycle_stats/h_mean
ref_surf/dem_h
ref_surf/geoid_h

Parameter minimum values, with one value per cycle, filled in NaN for cycles with no selected segments:

cycle_stats/cloud_flg_asr
cycle_stats/cloud_flg_atm
cycle_stats/bsnow_conf

Other parameters:

cycle_stats/strong_spot: The laser beam number for the strong beam in the pair

Algorithm:

1. Select the segments for the cycle indicated in *selected_segments* from the *ATL06_data_structure.*

2: Based on *h_li_sigma*, calculate the segment weights using equation 14.

3.1 For ATL06 parameters *h_robust_sprd, h_li_rms, r_eff, tide_ocean, dac, bsnow_h, x_atc, y_atc, sigma_geo_h, sigma_geo_at, sigma_geo_xt,* and *h_mean* calculate the weighted average of the parameter based on the segment weights. The output parameter names are the same as the input parameter names, in the cycle_stats group.
3.2 For ATL06 parameters *dem_h* and *geoid_h,* by regression between the measurement location and the reference point location. The output parameter names are the same as the input parameter names, in the *ref_surf* group.
4. For ATL06 parameters *cloud_flg_asr* and *cloud_flg_atm* report the best (minimum) value from among the selected values. For *bsnow_conf* report the maximum value from among the selected values.
5. For the cycle_stats/*strong_spot* attribute, report the laser beam number for the strong beam in the pair.

5.1.9 Characterize the reference surface

Inputs:

poly_coeffs: Coefficients of the surface polynomial

poly_coeff_sigma: Error estimates for the surface polynomial

degree_list_x, degree_list_y: exponents of the reference-surface polynomial for which the reference-surface fit returned a coefficient

rgt_azimuth: the azimuth of the reference ground track

Parameters:

poly_max_degree_AT, poly_max_degree_XT: Maximum polynomial degree allowed in x and y.

Outputs:

ref_surf/n_slope: the north component of the reference-surface slope

ref_surf/e_slope: the east component of the reference-surface slope

ref_surf/at_slope: the along-track component of the reference-surface slope

ref_surf/xt_slope: the across-track component of the reference-surface slope

ref_surf/rms_slope_fit: the rms slope of the reference surface

ref_surf/poly_ref_surf: the polynomial reference surface coefficients

ref_surf/poly_ref_surf_sigma: error estimates for *ref_surf/poly_ref_surf*

Procedure:

1. Calculate the coordinates of a grid of northing and easting offsets around the reference points, each between -50 m and 50 m in 10-meter increments: dN, dE

2. Translate the coordinates into along and across-track coordinates:

$$dx=\cos(rgt\_azimuth)*dN + \sin(rgt\_azimuth)*dE$$

$$dy=\sin(rgt\_azimuth)*dN - \cos(rgt\_azimuth)*dE$$

3. Calculate the polynomial surface elevations for the grid points by evaluating the polynomial surface at *dx* and *dy*: *z_poly*

4. Fit a plane to *z_poly* as a function of *dN* and *dE*. The North coefficient of the plane is *ref_surf/n_slope*, the east component is ref_surf/e_slope, the RMS misfit of the plane is *ref_surf/rms_slope_fit*. If either component of the slope has a magnitude larger than 0.2, add 2 to *ref_surf/fit_quality*.

5. Fit a plane to *z_poly* as a function of *dx* and *dy*. The along-track coefficient of the plane is *ref_surf/at_slope*, the across-track component is *ref_surf/xt_slope*.

6. Generate the polynomial exponents for the output columns. The list of components for the output variables has one component for each unique degree combination of *x* degrees between 0 and *ref_surf/deg_x* and for *y* degree between 0 and *ref_surf/deg_y* such that *x_degree + y_degree* <= *max(poly_max_degree_XT, poly_max_degree_AT)*, except that there is no *x_degree=0* and *y_degree=0* combination. They are sorted first by the sum of the *x* and *y* degrees, then by *x* degree, then by *y* degree.
Match the polynomial degrees for this reference point's coefficients to these degrees, and write each value of *poly_ref_surf* and *poly_ref_surf_sigma* into the appropriate position of the output array, filling missing values with *invalid*.

6.0 APPENDIX A: GLOSSARY

This appendix defines terms that are used in ATLAS ATBDs, as derived from a document circulated to the SDT, written by Tom Neumann. Some naming conventions are borrowed from **Spots, Channels and Redundancy Assignments** (ICESat-2-ATSYS-TN-0910) by P. Luers. Some conventions are different than those used by the ATLAS team for the purposes of making the data processing and interpretation simpler.

Spots. The ATLAS instrument creates six spots on the ground, three that are weak and three that are strong, where strong is defined as approximately four times brighter than weak. These designations apply to both the laser-illuminated spots and the instrument fields of view. The spots are numbered as shown in Figure 1. At times, the weak spots are leading (when the direction of travel is in the ATLAS +x direction) and at times the strong spots are leading. However, the spot number does not change based on the orientation of ATLAS. The spots are always numbered with 1L on the far left and 3R on the far right of the pattern. Not: beams, footprints.

Laser pulse (pulse for short). Individual pulses of light emitted from the ATLAS laser are called laser pulses. As the pulse passes through the ATLAS transmit optics, this single pulse is split into 6 individual transmit pulses by the diffractive optical element. The 6 pulses travel to the earth's surface (assuming ATLAS is pointed to the earth's surface). Some attributes of a laser pulse are the wavelength, pulse shape and duration. Not: transmit pulse, laser shot, laser fire.

Laser Beam. The sequential laser pulses emitted from the ATLAS instrument that illuminate spots on the earth's surface are called laser beams. ATLAS generates 6 laser beams. The laser beam numbering convention follows the ATLAS instrument convention with strong beams numbered 1, 3, and 5 and weak beams numbered 2, 4, and 6 as shown in the figures. Not: beamlet.

Transmit Pulse. Individual pulses of light emitted from the ICESat-2 observatory are called transmit pulses. The ATLAS instrument generates 6 transmit pulses of light from a single laser pulse. The transmit pulses generate 6 spots where the laser light illuminates the surface of the earth. Some attributes of a given transmit pulse are the wavelength, the shape, and the energy. Some attributes of the 6 transmit pulses may be different. Not: laser fire, shot, laser shot, laser pulse.

Reflected Pulse. Individual transmit pulses reflected off the surface of the earth and viewed by the ATLAS telescope are called reflected pulses. For a given transmit pulse, there may or may not be a reflected pulse. Not: received pulse, returned pulse.

Photon Event. Some of the energy in a reflected pulse passes through the ATLAS receiver optics and electronics. ATLAS detects and time tags some fraction of the photons that make up the reflected pulse, as well as background photons due to sunlight or instrument noise. Any photon that is time tagged by the ATLAS instrument is called a photon event, regardless of source. Not: received photon, detected photon.

Reference Ground Track (RGT). The reference ground track (RGT) is the track on the earth at which a specified unit vector within the observatory is pointed. Under nominal operating conditions, there will be no data collected along the RGT, as the RGT is spanned by GT2L and GT2R (which are not shown in the figures, but are similar to the GTs that are shown). During spacecraft slews or off pointing, it is possible that ground tracks may intersect the RGT. The precise unit vector has not yet been defined. The ICESat-2 mission has 1387 RGTs, numbered from 0001xx to 1387xx. The last two digits refer to the cycle number. Not: ground tracks, paths, sub-satellite track.

Cycle Number. Over 91 days, each of the 1387 RGTs will be targeted in the Polar Regions once. In subsequent 91-day periods, these RGTs will be targeted again. The cycle number tracks the number of 91-day periods that have elapsed since the ICESat-2 observatory entered the science orbit. The first 91-day cycle is numbered 01; the second 91-day cycle is 02, and so on. At the end of the first 3 years of operations, we expect the cycle number to be 12. The cycle number will be carried in the mid-latitudes, though the same RGTs will (in general) not be targeted more than once.

Sub-satellite Track (SST). The sub-satellite track (SST) is the time-ordered series of latitude and longitude points at the geodetic nadir of the ICESat-2 observatory. In order to protect the ATLAS detectors from damage due to specular returns, and the natural variation of the position of the observatory with respect to the RGT throughout the orbit, the SST is generally not the same as the RGT. Not: reference ground track, ground track.

Ground Tracks (GT). As ICESat-2 orbits the earths, sequential transmit pulses illuminate six ground tracks on the surface of the earth. The track width is approximately 10m wide. Each ground track is numbered, according to the laser spot number that generates a given ground track. Ground tracks are therefore always numbered with 1L on the far left of the spot pattern and 3R on the far right of the spot pattern. Not: tracks, paths, reference ground tracks, footpaths.

Reference Pair Track (RPT). The reference pair track is the imaginary line halfway between the planned locations of the strong and weak ground tracks that make up a pair. There are three RPTs: RPT1 is spanned by GT1L and GT1R, RPT2 is spanned by GT2L and GT2R (and may be coincident with the RGT at times), and RPT3 is spanned by GT3L and GT3R. Note that this is the planned location of the midway point between GTs. We will not know this location very precisely prior to launch. Not: tracks, paths, reference ground tracks, footpaths, pair tracks.

Pair Track (PT). The pair track is the imaginary line half way between the actual locations of the strong and weak ground tracks that make up a pair. There are three PTs: PT1 is spanned by GT1L and GT1R, PT2 is spanned by GT2L and GT2R (and may be coincident with the RGT at times), and PT3 is spanned by GT3L and GT3R. Note that this is the actual location of the midway point between GTs, and will be defined by the actual location of the GTs. Not: tracks, paths, reference ground tracks, footpaths, reference pair tracks.

Pairs. When considered together, individual strong and weak ground tracks form a pair. For example, GT2L and GT2R form the central pair of the array. The pairs are numbered 1 through 3: Pair 1 is comprised of GT1L and GT1R, pair 2 is comprised of GT2L and GT2R, and pair 3 is comprised of GT3L and 3R.

Along-track. The direction of travel of the ICESat-2 observatory in the orbit frame is defined as the along-track coordinate, and is denoted as the +x direction. The positive x direction is therefore along the Earth-Centered Earth-Fixed velocity vector of the observatory. Each pair has a unique coordinate system, with the +x direction aligned with the Reference Pair Tracks.

Across-track. The across-track coordinate is y and is positive to the left, with the origins at the Reference Pair Tracks.

Segment. An along-track span (or aggregation) of PE data from a single ground track or other defined track is called a segment. A segment can be measured as a time duration (e.g. from the time of the first PE to the time of the last PE), as a distance (e.g. the distance between the location of the first and last PEs), or as an accumulation of a desired number of photons. Segments can be as short or as long as desired.

Signal Photon. Any photon event that an algorithm determines to be part of the reflected pulse.

Background Photon. Any photon event that is not classified as a signal photon is classified as a background photon. Background photons could be due to noise in the ATLAS instrument (e.g. stray light, or detector dark counts), sunlight, or mis-classified signal photons. Not: noise photon.

h_.** Signal photons will be used by higher-level products to determine height above the WGS-84 reference ellipsoid, using a semi-major axis (equatorial radius) of 6378137m and a flattening of 1/298.257223563. This can be abbreviated as ‘ellipsoidal height’ or ‘height above ellipsoid’. These heights are denoted by h; the subscript ** will refer to the specific algorithm used to determine that elevation (e.g. is = ice sheet algorithm, si = sea ice algorithm, etc…). Not: elevation.

Photon Cloud. The collection of all telemetered photon time tags in a given segment is the (or a) photon cloud. Not: point cloud.

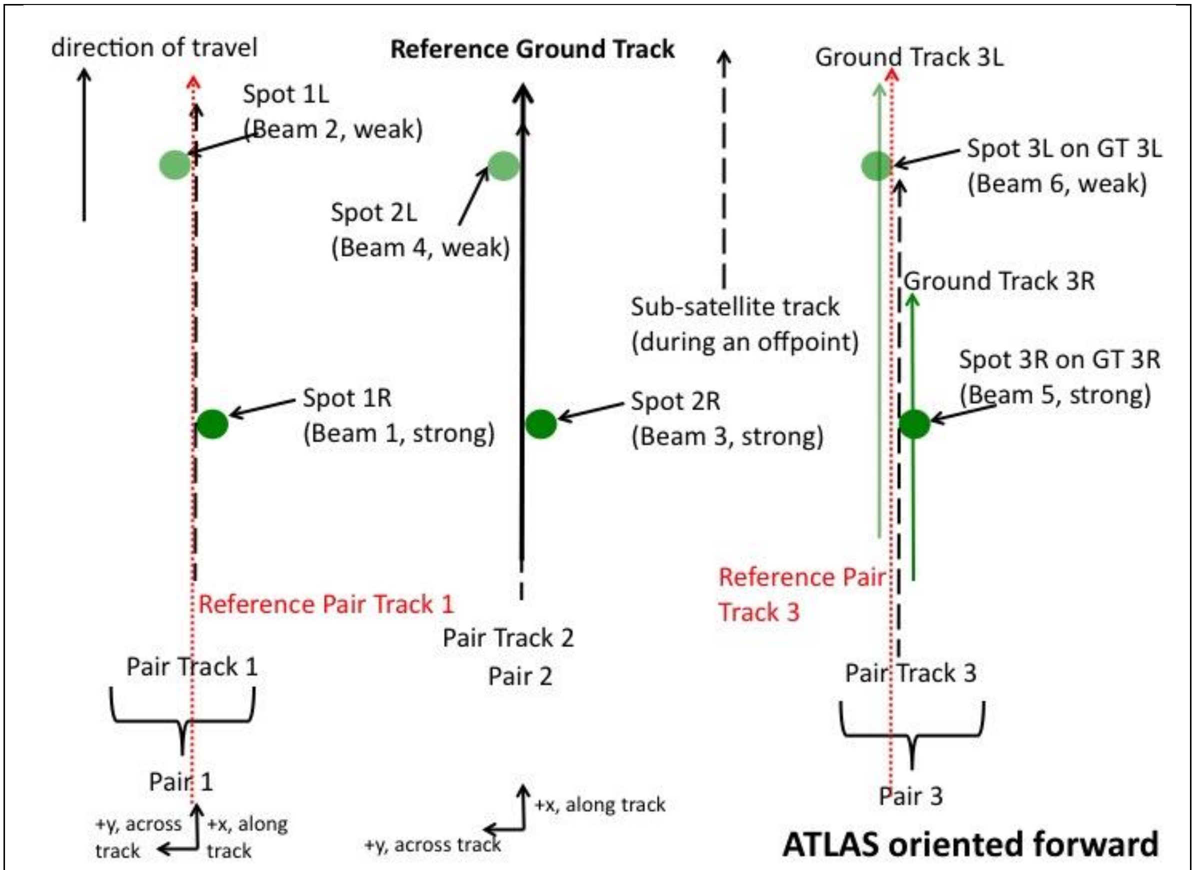
Background Count Rate. The number of background photons in a given time span is the background count rate. Therefore a value of the background count rate requires a segment of PEs and an algorithm to distinguish signal and background photons. Not: Noise rate, background rate.

Noise Count Rate. The rate at which the ATLAS instrument receives photons in the absence of any light entering the ATLAS telescope or receiver optics. The noise count rate includes PEs due to detector dark counts or stray light from within the instrument. Not: noise rate, background rate, and background count rate.

Telemetry band. The subset of PEs selected by the science algorithm on board ATLAS to be telemetered to the ground is called the telemetry band. The width of the telemetry band is a function of the signal to noise ratio of the data (calculated by the science algorithm onboard ATLAS), the location on the earth (e.g. ocean, land, sea ice, etc…), and the roughness of the terrain, among other parameters. The widths of telemetry bands are adjustable on-orbit. The telemetry bandwidth is described in Section 7 or the ATLAS Flight Science Receiver Algorithms document. The total volume of telemetered photon events must meet the data volume constraint (currently 577 GBits/day).

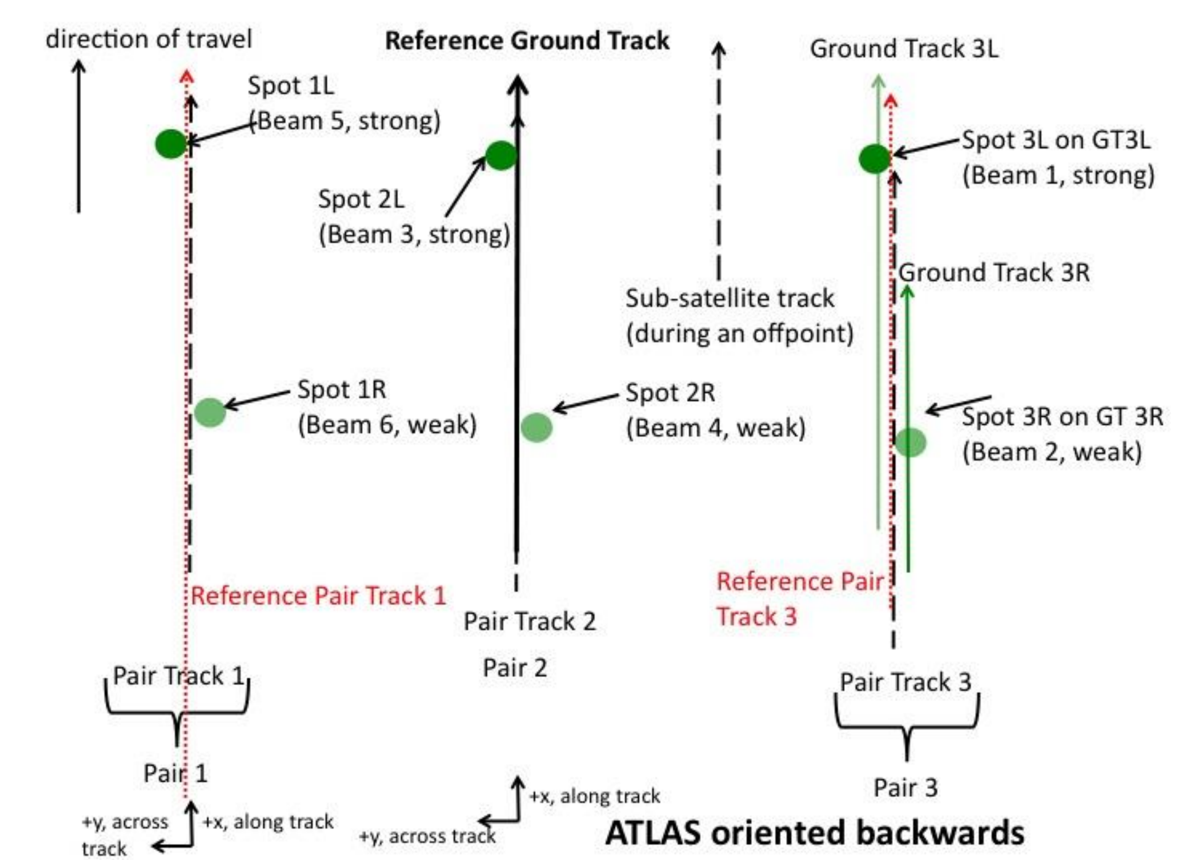
Window, Window Width, Window Duration. A subset of the telemetry band of PEs is called a window. If the vertical extent of a window is defined in terms of distance, the window is said to have a width. If the vertical extent of a window is defined in terms of time, the window is said to have a duration. The window width is always less than or equal to the telemetry band.

Figure 6-1. Spots and tracks, forward flight



Spot and track naming convention with ATLAS oriented in the forward (instrument coordinate +x) direction.

Figure 6-2. Spots and tracks, backward flight



Spot and track naming convention with ATLAS oriented in the backward (instrument coordinate -x) direction.

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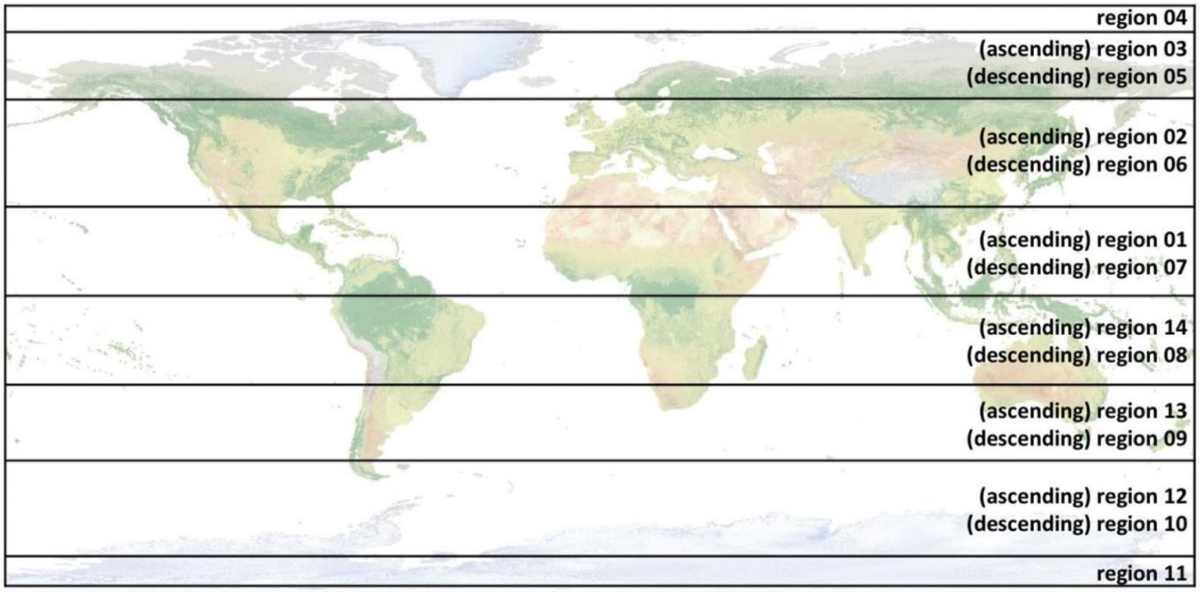
Figure 6-3. Granule regions



Along-track product granule regions

7.0 BROWSE PRODUCTS

For each ATL11 data file, there will be eight figures written to an associated browse file. Two of these figures are required and are located in the default group; default1 and default2. The browse filename has the same pattern as the data filename, namely, ATL11_ttttss_c1c2_rr_vVVV_BRW.h5, where tttt is the reference ground track, ss is the orbital segment, c1 is the first cycle of data in the file, c2 is the last cycle of data in the file, rr is the release and VVV is the version. Optionally, the figures can also be written to a pdf file.

Below is a discussion of the how the figures are made, with examples from the data file ATL11_009403_0307_02_vU07.h5. Note that the figure numbering in this section is distinct from that in the rest of the document; the figures shown here are labeled as they are in each browse-product file.

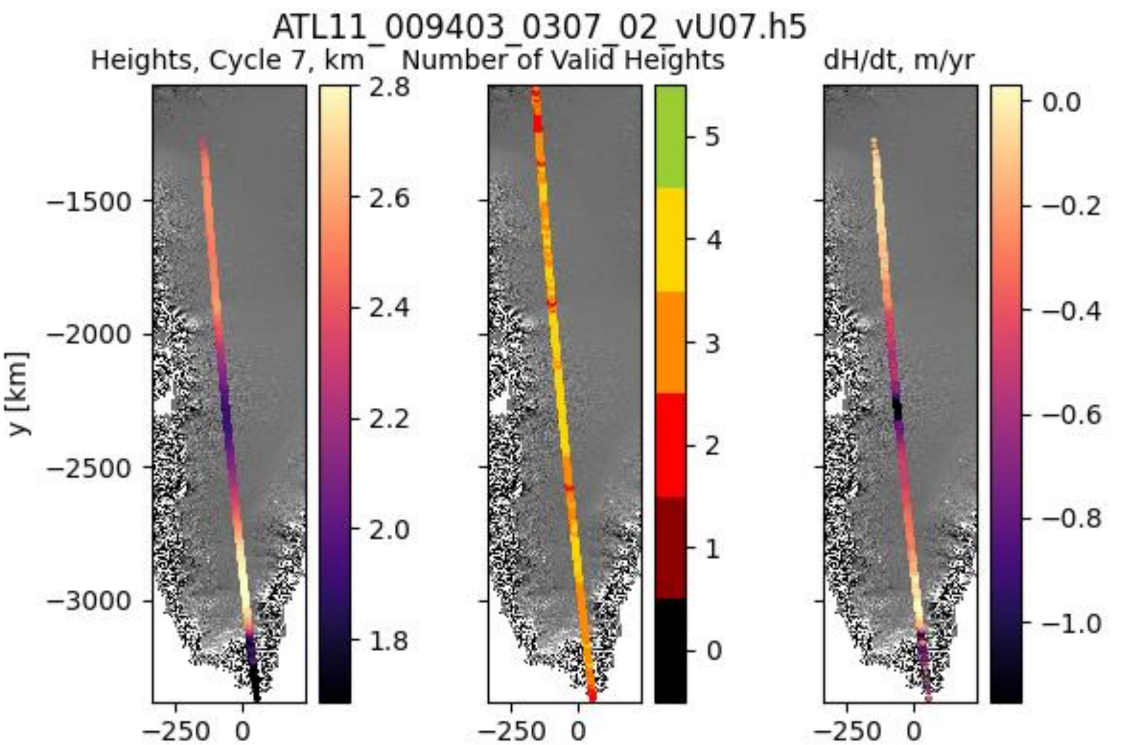


Figure 1. Height data, in km, from cycle 7 (1st panel). Number of cycles with valid height data (2nd panel). Change in height over time, in meters/year, cycle 7 from cycle 3 (3rd panel). All overlaid on gradient of DEM. x, y in km. Maps are plotted in a polar-stereographic projection with a central longitude of 45W and a standard latitude of 70N.

The background for the three panels in Figure 1 is the gradient DEM in gray scale. It is shown in a polar-stereographic projection with a central longitude of 45W (0E) and a standard latitude of

70N (71S), for the Northern (Southern) Hemisphere. The map is bounded by the extent of height data plus a buffer. ATL11 heights (/ptx/h_corr) from all pairs of the latest cycle with valid data, here cycle 7, are plotted in the first panel. The “magma” color map indicates the heights in km. The limits on the color bar are set with the python scipy.stat.scoreatpercentile method at 5% and 95%. In the second panel are plotted the number of valid heights summed over all cycles at each location. The color bar extends to the total number of cycles in the data file. The change in height over time, dH/dt, is plotted in the third panel, in meters/year. dHdt is the change in height of the last cycle with valid data from the first cycle with valid data (/ptx/h_corr) divided by the associated times (/ptx/delta_time). Text of ‘No Data’ is printed in the panel if there is only one cycle with valid data, or if the first and last cycles with valid data have no common reference point numbers (/ptx/ref_pt). All plots are in x,y coordinates, in km. This figure is called default/default1 in the BRW.h5 file.

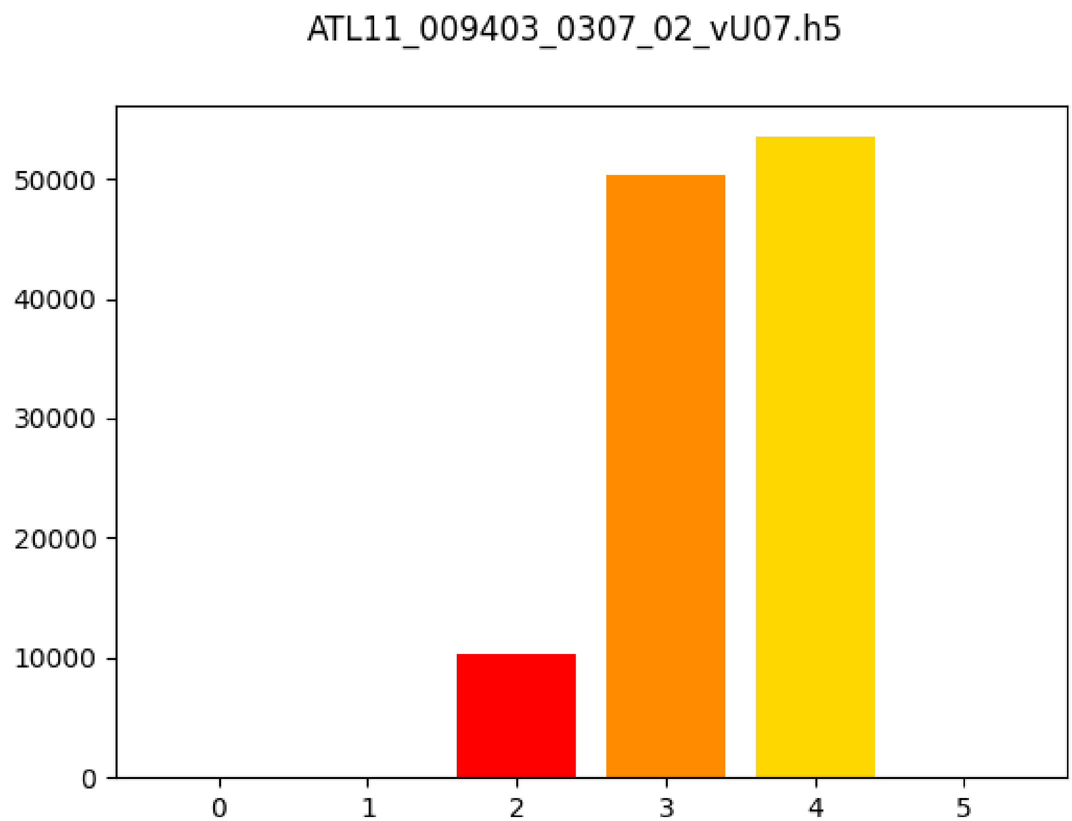


Figure 2. Histogram of number of cycles with valid height measurements, all beam pairs.

A histogram of the number of valid height measurements (/ptx/h_corr) is in Figure 2. Valid height data are summed across all cycles, for each reference point number (/ptx/ref_pt). The color scale is from zero to the total number of cycles in the data file and matches those in Figure 1, 2nd panel. This figure is called validrepeats_hist in the BRW.h5 file.

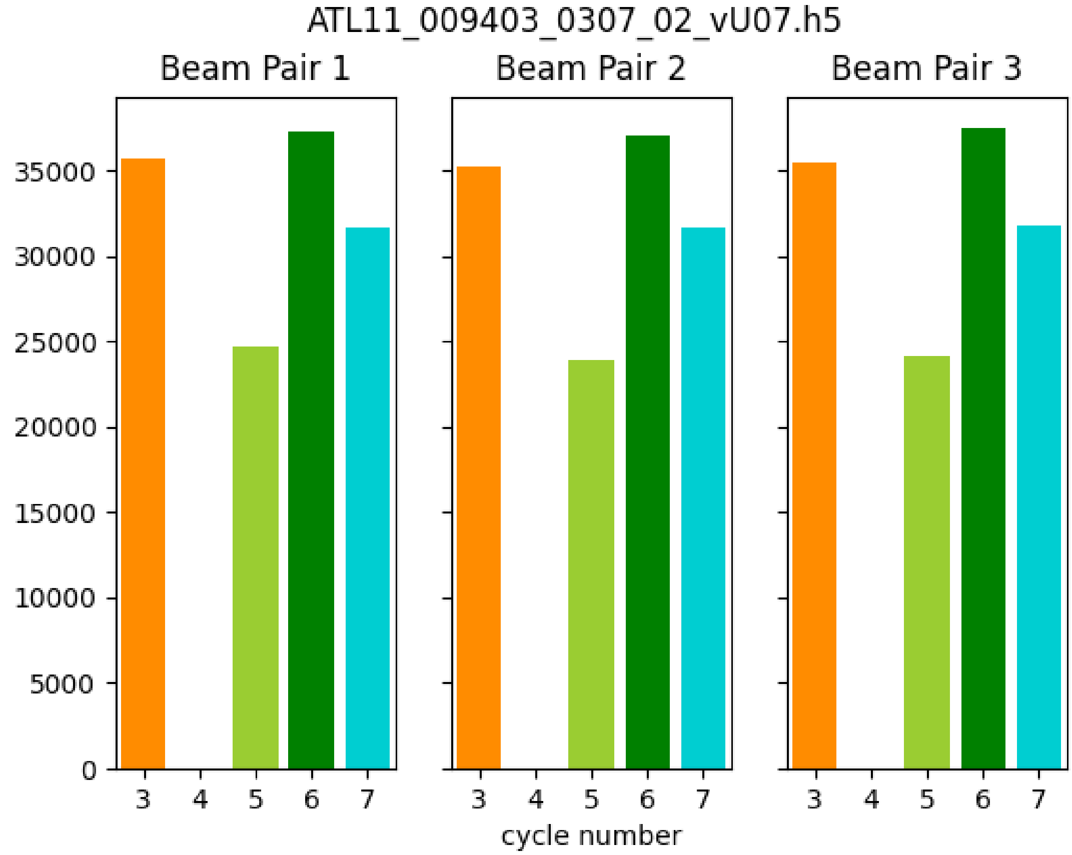


Figure 3. Number of valid height measurements from each beam pair.

Histograms in Figure 3 show the number of valid heights (/ptx/h_corr) for each cycle, separated by beam pair. The cycle numbers are color coded. This figure is called default/default2 in the BRW.h5 file.

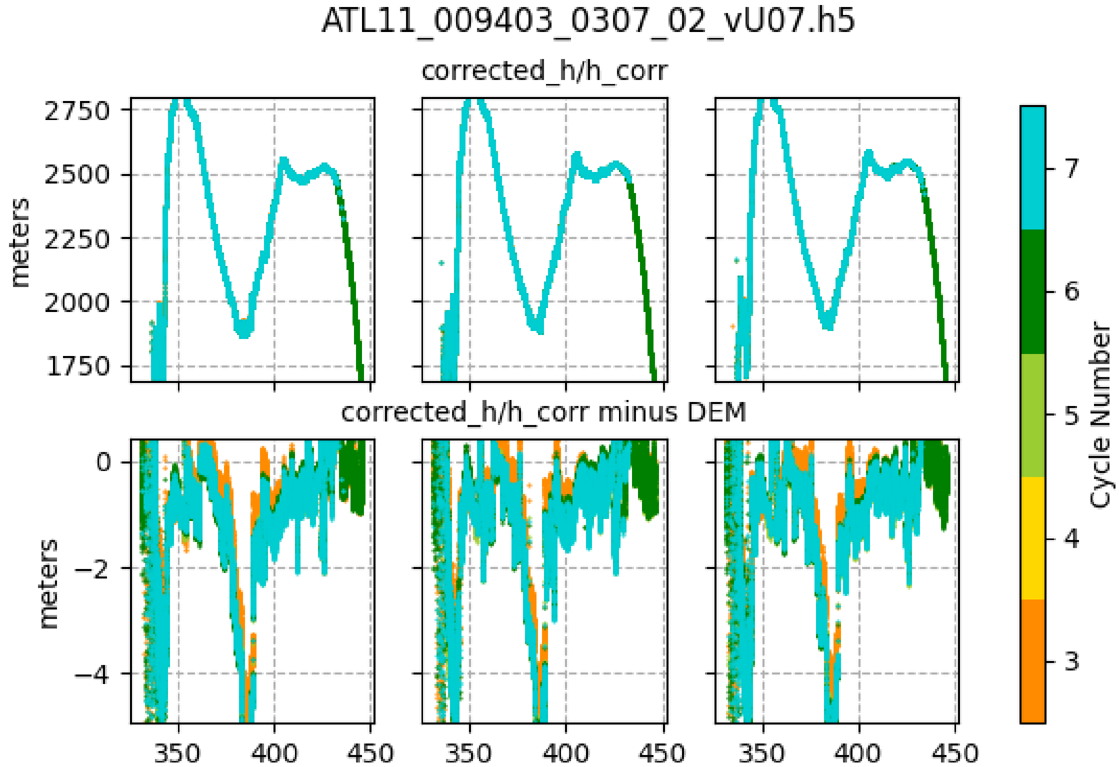


Figure 4. Top row: Heights, in meters, plotted for each beam pair: 1 (left), 2 (center), 3 (right). Bottom row: Heights minus DEM, in meters. Y-axis limits are scores at 5% and 95%. Color coded by cycle number. Plotted against reference point number/1000.

There are six panels in Figure 4, with two rows and three columns. In the top row are plotted the height measurements (/ptx/h_corr) for each beam pair, one pair per panel. In the bottom row are plotted the same height measurements minus the collocated DEM (ref_surf/dem_h) values, one pair per panel. The plots are color coded by cycle number, as in Figure 3. The heights are plotted versus reference point number (/ptx/ref_pt) divided by 1000 for a cleaner plot. The y-axis is in meters for both rows. The y-axis limits for the top and bottom rows are set separately, using the python scipy.stats.scoreatpercentile method with limits of 5% and 95% for heights and height differences, respectively. Text of ‘No Data’ is printed in a panel if there are no valid height data for that pair. This figure is called h_corr_h_corr-DEM in the BRW.h5 file.

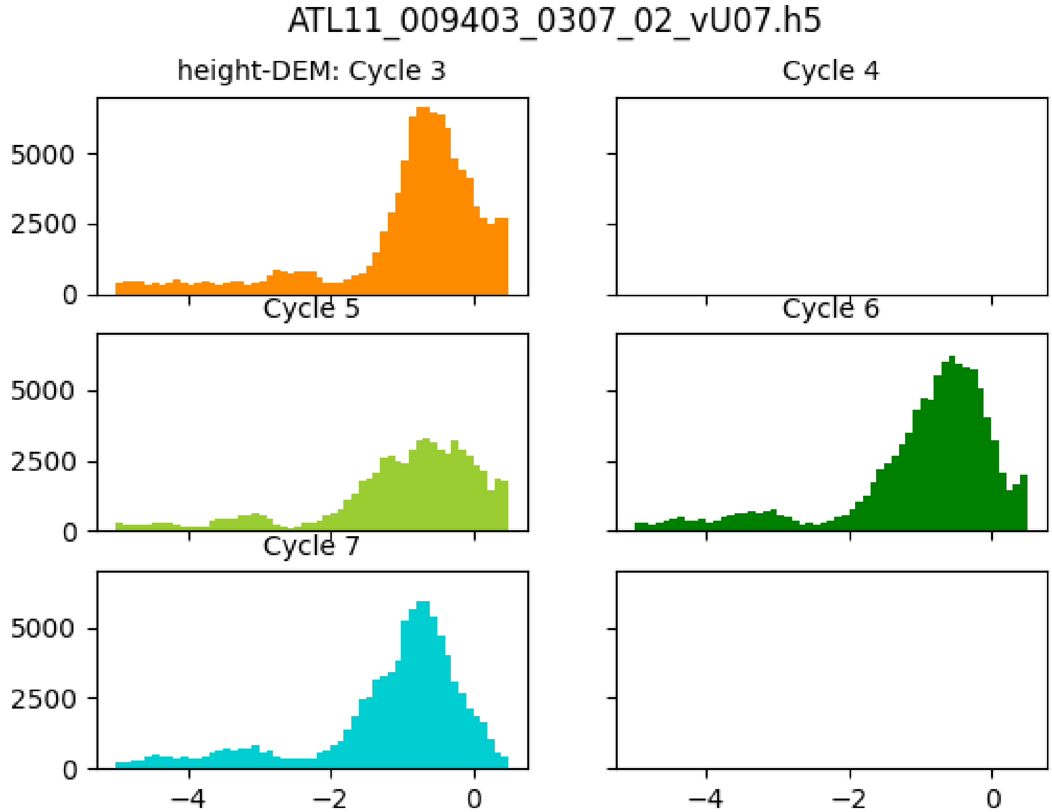


Figure 5. Histograms of heights minus DEM heights, in meters. One histogram per cycle, all beam pairs. X-axis limits are the scores at 5% and 95%.

Figure 5 is associated with Figure 4. It is a multi-paneled figure, with the number of panels dependent on the number of cycles in the data file. Each panel is a histogram of the heights (/ptx/h_corr) minus collocated DEM heights (ref_surf/dem_h) color coded by cycle, the same as in Figures 3 and 4. The limits on the histograms are set using the python scipy.stats.scoreatpercentile method with limits of 5 and 95% for all cycles of data, the same values used in Figure 4 bottom row. This figure is called h_corr-DEM_hist in the BRW.h5 file.

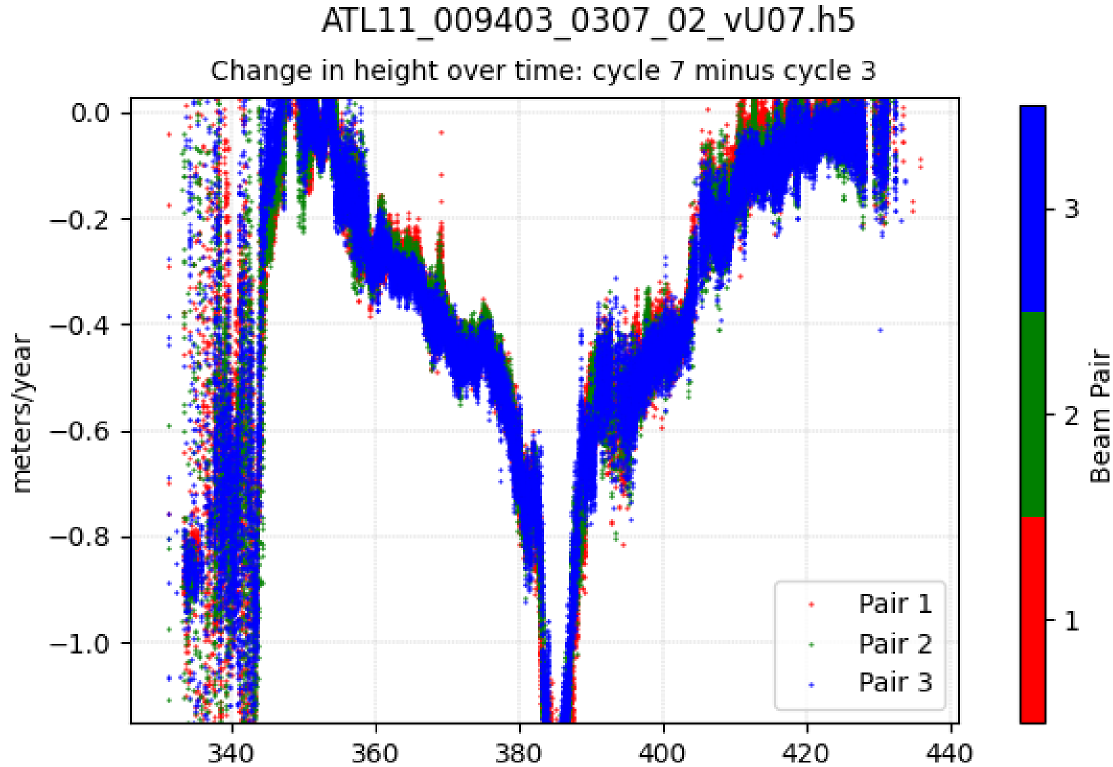


Figure 6. Change in height over time, dH/dt, in meters/year. dH/dt is cycle 7 from cycle 3. Color coded by beam pair: 1 (red), 2 (green), 3 (blue). Y-axis limits are scores at 5% and 95%. Plotted against reference point number/1000.

The changes in height with time, dH/dt, in meters/year are plotted in Figure 6. The calculation differences the first and last cycles with valid height data (/ptx/h_corr) divided by the associated time differences (/ptx/delta_time). The change in heights for pair 1 are in red, for pair 2 are in green and for pair 3 are in blue. The y-axis limits are set using the python scipy.stats.scoreatpercentile method with limits of 5% and 95%. The x-axis is reference point number (/ptx/ref_pt) divided by 1000 for a cleaner plot. Text of 'No Data' is printed in the panel if there is only one cycle with valid data, or if the first and last cycles with valid data have no common reference point numbers. This figure is called dHdt in the BRW.h5 file.

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ASAS	ATLAS Science Algorithm Software
ATBD	Algorithm Theoretical Basis Document
ATLAS	ATLAS Advance Topographic Laser Altimeter System
CDF	Cumulative Distribution Function
DEM	Digital Elevation Model
GSFC	Goddard Space Flight Center
GTs	Ground Tracks
ICESat-2	Ice, Cloud, and Land Elevation Satellite-2
IKR	I Know, Right?
MABEL	Multiple altimeter Beam Experimental Lidar
MIS	Management Information System
NASA	National Aeronautics and Space Administration
PE	Photon Event
POD	Precision Orbit Determination
PPD	Precision Pointing Determination
PRD	Precise Range Determination
PSO	ICESat-2 Project Science Office
PTs	Pair Tracks
RDE	Robust Dispersion Estimate
RGT	Reference Ground Track
RMS	Root Mean Square
RPTs	Reference Pair Tracks
RT	Real Time
SCoRe	Signature Controlled Request
SIPS	ICESat-2 Science Investigator-led Processing System
TLDR	Too Long, Didn't Read
TBD	To Be Determined

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